Theorizing Trikāla: A Generalized Intervallic Approach to Pulse Transformation in South Indian Carnatic Music[1]

Robert Wells

SOUTH Indian Carnatic (classical) music, a centuries-old musical tradition grounded in Hindu spiritual devotion, is built upon incredibly intricate rhythmic/metric foundations. In this music, the constant flow of time is represented by an internalized metric cycle called the *tāḷa*, consisting of beats (*akṣara*s) that are grouped into larger units: *āvarta*s, which resemble Western measures and consist of single cycles of tāḷa, and *anga*s, which are characteristic subdivisions of each āvarta. Moreover, performers and informed listeners track location in the tāḷa using *kriyā*s, which are standardized hand gestures that include claps, waves, and finger counts.

Figure 1 illustrates these concepts with respect to Ādi tāḷa, the most common Carnatic tāḷa. In this diagram, observe that there are eight beats per tāḷa cycle (āvarta), and each cycle is subdivided as 4 + 2 + 2, yielding three angas. Moreover, each anga corresponds to a specific set of kriyās, as illustrated in the figure.

While a soloist's melodic/rhythmic phrasing in a Carnatic performance often reinforces the underlying tāḷa through characteristic gestures and rhythmic patterns, these phrases may also express temporary tension with the tāḷa.[2] A central Carnatic improvisational form that exploits both types of relationship with the tāḷa is the three-part *rāgam-tānam-pallavi* form,

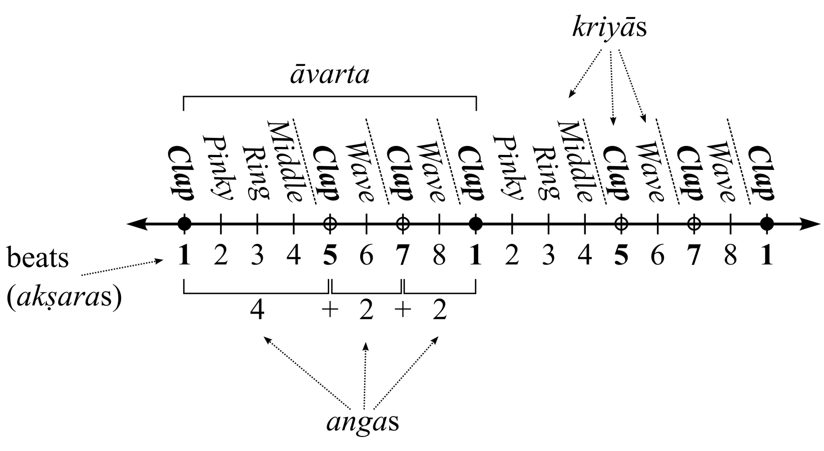


Figure 1. Basic Carnatic rhythmic/metric concepts in Ādi tāḷa (4 + 2 + 2).

1. This is a significantly expanded version of a paper presented at the Fourth International Conference on Analytical Approaches to World Music held in New York City, USA, June 8–11, 2016.
2. Carnatic musicians use the term *sarvalaghu* ("time flow") to describe musical gestures and patterns that reinforce the tāḷa and *kaṇakku* ("calculation") to refer to the mathematical planning necessary to execute patterns that create tension with the tāḷa. See Nelson (2000, 153–56).

often abbreviated as "RTP." The rāgam section that opens RTP performances is an unmetered melodic improvisation in free rhythm that introduces the primary *rāga*, or melodic mode.[3] The tānam section also consists of unmetered melodic improvisation, but the soloist now incorporates a rhythmic pulse. In the case of vocal RTP performances, this pulse is generated by percussive vocal syllables. Finally, the pallavi section comprises a set of improvised, tāḷa-bound variations on a short line of melody, which is generally texted even if the variants are not.

While the pallavi section progresses through several distinct improvisational stages, each with characteristic parameters and expectations, this study will primarily investigate a variation technique known as *trikāla*.[4] This performance technique challenges the listener's sense of tāḷa while demonstrating the soloist's control over musical time. In trikāla, the pallavi melody (or other Carnatic melody) is presented in several different speeds over the constant tāḷa, and while this traditionally involves three speeds (original speed, double or half speed, and quadruple or quarter speed), a performer may exploit many more speeds than this.[5] An important question, then, is how trikāla can function locally and globally to generate tension with the tāḷa and shape rhythmic flow in RTP and other Carnatic forms. The current article seeks to investigate this question in a quantitative manner that will allow precise, dynamic relationships to come to the surface in trikāla-based musical excerpts.

To see how trikāla can function in Carnatic music, consider an RTP performance by T. Viswanathan, L. Shankar, and T. Ranganathan (1973). The pallavi is in the asymmetrical Miṣra Cāpu tāḷa, which consists of seven quick beats partitioned as 3 + 2 + 2.[6] This pallavi contains a remarkable instance of trikāla technique, as Widdess (1977) has previously noted.[7] Figure 2 illustrates the main pallavi melody, which may be heard in Audio Example 1.

Because the current study's analytical focus will be on rhythm and meter, I have not included a detailed depiction of the melodic ornaments (*gamaka*s), but have limited pitch content to Widdess's melodic simplification.[8] Unlike Widdess, though, I have chosen to bar the

3. Indian rāgas are not merely series of pitches within the octave, but are characterized by specialized sets of ornaments (*gamaka*s), characteristic phrases, and even extramusical ideas. See Viswanathan and Cormack (1998), Morris (2006, 307–8), and Schachter (2015).
4. In addition to the trikāla section, which forms this paper's focus, standard pallavi sections include the *niraval*, consisting of improvisations that maintain the basic rhythmic structure of the pallavi; the *swara kalpana*, whose improvisations employ sargam, or melodic solfège syllables; and the *ragamalika* ("garland of ragas"), in which the performer improvises in a series of different ragas. For more on these pallavi stages, see Subramaniam and Subramaniam (1995, 87–88), Krishna (2013, 160–61), and Field (2018).
5. For a particularly virtuosic example, see Subramaniam and Iyer's performance of the varṇam *Jalasaksha* (rāga Hamsadhwani; Ādi tāḷa) on Subramaniam and Iyer (2013). The recording can also be heard at https://youtu.be/DqxFpF4R-Go. In this performance, the opening melody is presented in no fewer than thirteen different speeds. I thank Michael Tenzer for bringing this example to my attention.
6. The kriyās consist of claps with the back of the hand on beats 1 and 2 and palm claps on beats 4 and 6.
7. The trikāla section comprises the final three minutes of Viswanathan, Shankar, and Ranganathan (1973), Side 1.
8. Regarding pitch, the improvisation is in rāga Shaṅkarābharaṇam, whose unadorned form (without gamakas) is analogous to the Western major scale.

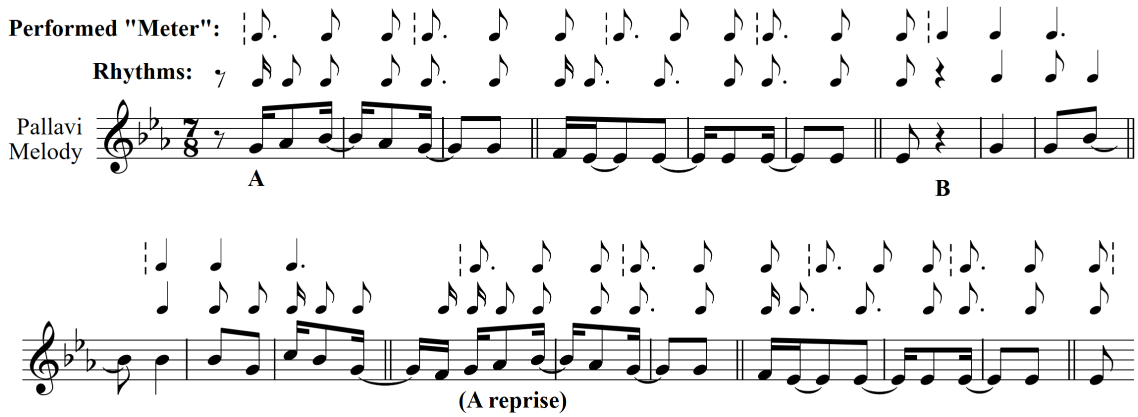


Figure 2. RTP performance by T. Viswanathan, L. Shankar, and T. Ranganathan (1973), pallavi melody. Transcription after Widdess (1977), with metric annotations by the author.

Audio Example 1. Pallavi melody from RTP performance by T. Viswanathan, L. Shankar, and T. Ranganathan (1973), with referential chimes added to mark the first beat of each tāḷa cycle.

melody with respect to the tāḷa and angas, with the melody starting on beat 2.[9] Specifically, solid double bar lines mark out full tāḷa cycles (āvartas), while solid single bar lines demarcate angas. Above the staff, I represent the rhythms of the melody without ties or bar lines to facilitate identifying rhythmic patterns and groupings. Above this layer, I have provided a more flexible "barring" corresponding to Widdess's suggested metric units within the melody. Finally, I have included labels corresponding to Widdess's sectioning of the melody, consisting of an opening A phrase, a B phrase with a slower pulse, and a reprise of A (63).

Figure 3 presents the opening portion of the trikāla section, which progressively augments and diminishes the pallavi melody over constant tāḷa while incorporating subtle variations. In this lengthier transcription, double dotted bar lines in the "Performed 'Meter'" layer bound complete A–B melodic statements.[10] Observe that the pulse units in this layer expand well beyond what occurred in the original melody, as illustrated in Audio Example 2. Additionally, note that the performed "meter" alternates between apparent 2+2+3 subdivisions for the A section of the melody and 3+2+2 for the B section; thus, not only are the magnitudes of the melodic pulse units flexible, but also the pulse units' arrangements.[11]

9. Because the melody consistently begins on beat 2 of the tāḷa throughout the trikāla section and all changes of pulse unit start from this beat (see Figure 3), Widdess (1977, 63) bars his transcription so that the second beat of the tāḷa is "beat one" and the basic subdivisions of the tāḷa are 2+2+2+1 rather than 3+2+2. This decision is largely a matter of convenience, as Widdess represents changes of pulse using time signatures, which are cleaner to notate at the beginnings of measures than mid-measure.
10. Note that the excerpt opens with a B statement. As Widdess (1977, 64) points out, this partial melodic statement is completed by a final A statement at the end of the trikāla section.
11. See Widdess (1977, 63–64) regarding the metric subdivisions of the pallavi. Additionally, Subramaniam and Subramaniam (1995, 75) point out that the seven beats of Miṣra Cāpu tāḷa may be grouped as 3+2+2 *or* 2+2+3, which

Figure 3. RTP performance by T. Viswanathan, L. Shankar, and T. Ranganathan (1973), opening of trikāla section. Transcription after Widdess (1977), with metric annotations by the author.

Audio Example 2. RTP performance (Viswanathan, Shankar, and Ranganathan 1973), opening of trikala section.

Widdess makes fascinating observations about the effect of the trikāla usage here, pointing out the various "tempos" in which the pallavi melody is presented (62) and mentioning the evolving misalignment of claps with the melody (66).[12] How, though, might one achieve the aforementioned goal of making such statements more precise and quantifiable, and delve even deeper into the tensions between melody and tāḷa in trikāla-based performance? More specifically, how might expansion and contraction processes in Carnatic music be modeled numerically?

In addition to Widdess's study, numerous others have investigated tāḷa and rhythmic/metric conflict in Carnatic music in varied ways. Morris has discussed tensions between characteristic rhythmic cadences and the underlying tāḷa in Carnatic performances (2000), as well as classifications of tāḷas based on their internal properties (1998), although trikāla technique is not a primary focus of his work. Subramanian, Wyse, and McGee (2011) and Tenzer (2011) emphasize trikāla technique more heavily, the former investigating the effect

suggests that the alternating subdivisions in the "performed" meter reflect properties of the underlying tāḷa.
12. The claps in the recording mark the beginnings of the second and third angas of the tāḷa—i.e., beats 4 and 6.

of performed speed on melodic ornamentation and the latter comparing augmentation-based processes in Carnatic to those in other musics. Various other sources provide useful theoretical, cultural, historical, and philosophical insights into South Indian conceptions of rhythm and time (e.g., Śarma 1992; Subramaniam and Subramaniam 1995; Ramanathan and Venkataram 1997; Krishna 2013), including discussions of trikāla technique from a broader perspective.

While the above studies approach trikāla from a theoretical or analytical standpoint, other sources take a more performative slant. Nelson, for instance, not only provides detailed analyses of trikāla-based Carnatic rhythmic/metric structures, but relates these structures to experienced mridangam players' creative processes (1991) and presents techniques for vocally performing these structures (2008; 2019). Iyer (1988) emphasizes performance and improvisation to an even greater degree. In addition to discussing how to construct sophisticated rhythmic phrases within various tāḷas, he provides practical techniques and formulas for determining when to begin a phrase so that it ends at a desired point in the tāḷa (e.g., *sam*, the first beat of a tāḷa cycle).

The current study seeks to expand upon the previous studies' insights by bringing trikāla technique more squarely into the realm of contemporary mathematical music theory, where it may be examined in a new light. This said, the study is certainly not an attempt to "replace" traditional methods of Carnatic music, which facilitate extremely high levels of rhythmic complexity. Instead, the article seeks to provide useful tools of an analytical, rather than performative, nature. To express the dynamic and quantitative properties of South Indian metric structures, I adopt a theoretical approach based on David Lewin's concept of the *generalized interval system*, or *GIS*, discussed in *Generalized Musical Intervals and Transformations* (1987)—henceforth, *GMIT*. Intuitively, a GIS is a musical space in which one can measure abstract "intervals" satisfying special mathematical properties.[13]

Significantly, nothing in Lewin's GIS definition requires that intervals be measured between pitches or pitch classes. Indeed, many of Lewin's own examples in *GMIT* consider harmonic or rhythmic spaces.[14] For the current project, I will draw upon the GIS *Met* (Wells 2015a; 2015b; 2017), a *metric* GIS derived from conflicting metric layers. Though *Met* was primarily developed to model evolving conflict between metric layers in Western music, it is easily adaptable to Carnatic rhythmic/metric processes.

In the next section of this article, then, I introduce the GIS *Met* and demonstrate its applications to pulse expansion and contraction processes in Carnatic music. The following section will then apply these ideas to analyses of several traditional melodic exercises called *alankārams*, which are typically performed in three or four speeds over constant tāḷa. In the final main section of the article, I revisit the RTP performance discussed by Widdess and

13. A brief introduction to GIS theory appears in the next section. For a full, formal definition of the GIS, see Lewin's (1987, 26) Definition 2.3.1.
14. See, for instance, Lewin's (1987, 22–25; 60–81) Examples 2.2.1–2.2.4, 2.2.6, and the "time span GIS."

suggest new perspectives and analytical insights into the evolving tension between surface melodic phrasing and the underlying tāḷa. Beyond what this analysis will reveal about the RTP performance in question, the analytical techniques employed will have noteworthy implications for rhythmic/metric analysis of Carnatic music more generally.

THEORETICAL FRAMEWORK: TRIKĀLA AND *MET*

Deeper consideration of the rhythmic/metric subtleties of examples like the aforementioned RTP performance necessitates laying the theoretical groundwork, beginning with Lewin's notion of the generalized interval system (GIS). To illustrate this abstract theoretical structure, Figure 4 depicts the GIS from Lewin's (1987, 17) Example 2.1.2, which measures directed semitone distances between equally tempered chromatic pitches.

Appearing in the figure are the three essential components of any GIS: a GIS "space" (*S*), a mathematical group of "intervals" (*IVLS*),[15] and an "interval function" (*int*) that maps pairs of elements from *S* into *IVLS*. The space, in this case, is the infinite chromatic gamut, represented by an infinite keyboard. The *int* function measures intervals by counting the number of semitones, in a positive or negative direction, from one pitch to another. The result is an integer-valued interval, which may be added to other integer-valued intervals.

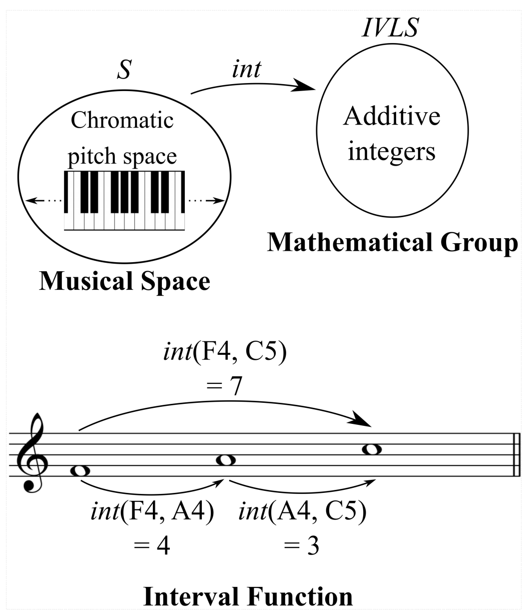


Figure 4. Illustration of the Lewinian GIS for calculating semitone intervals between pitches.

15. A group is a mathematical set endowed with an operation (+, ×, etc.) satisfying several properties. First, operating on a pair of elements yields an element that is still in the original set (closure). Moreover, the group must contain an identity element, and each element of the group must have an inverse. Finally, the group operation must be associative. See Fraleigh and Katz (2003, 37–38).

The bottom portion of Figure 4 illustrates, more specifically, how intervals can be measured and combined. To calculate the interval from F4 to A4, for instance, one may count half steps: *int*(F4, A4) = 4 semitones. Similarly, *int*(A4, C5) = 3 semitones. The wider, 7-semitone interval from F4 to C5 illustrates an important condition that all GISs must satisfy: the interval from F4 to C5 is the sum of *int*(F4, A4) and *int*(A4, C5). In general, for any pitches *r*, *s*, and *t* in *S*, Lewin (1987, 26) requires that *int*(*r*, *s*) + *int*(*s*, *t*) = *int*(*r*, *t*). The figure demonstrates another necessary GIS property as well: assume a starting pitch *s* = F4 and interval *i* = 4. Then if one begins at *s* and traverses the interval *i*, the result is a unique pitch, *t* = A4. In general, for any point *s* and interval *i*, there must exist a unique *t* lying the interval *i* from *s*.[16] In other words, a single interval from *s* cannot lead to two different places in the GIS.

The GIS *Met*, which will form the basis for this study's metric analyses of Carnatic music, is of a different sort, with points and intervals modeling time rather than pitch. The foundation of *Met*, as described by Wells (2015a; 2017), is an essential conflict between two metric layers: the *X-layer*, which is a referential, cognitively internalized metric layer, and the *Y-layer*, which is a heard or sonically articulated metric layer. For instance, in Western common-practice music, the notated barring might correspond to the *X*-layer while a conflicting heard meter could form the *Y*-layer. In Carnatic music, the underlying tāḷa might correspond to the *X*-layer, while a soloist's rhythmic-melodic phrasing could define the *Y*-layer.[17]

A quintessential example of conflicting *X*- and *Y*-layers appears in Figure 5, from the Gigue from J. S. Bach's English Suite No. 5 in E minor, BWV 810. This excerpt features a two-measure hemiola (mm. 42–43), in which two bars of notated 3/8 sound like three bars of 2/8, as indicated by the dotted bar lines. The excerpt can be heard in Audio Example 3. Because of the hemiola, the solid arrow above the score spans two heard "2/8" bars that correspond to 1 1/3 *notated* 3/8 bars. Consequently, the heard downbeat shifts from beat 1 of the notated measure to beat 2—a shift by one eighth note. These three pieces of information—number of notated measures (*X*-measures) spanned, number of heard measures (*Y*-measures) spanned, and shift of heard downbeat (*Y*-downbeat shift)—form the heart of the GIS *Met*.

The numerical and graphic annotations in Figure 5 provide a more detailed depiction of the workings of *Met*. In the figure, the aforementioned dotted bar lines mark out the local *Y*-meter of 2 (with an eighth-note pulse), while the *X*-meter, represented by the solid bar lines, maintains a constant value of 3.[18] Below the score are time point spaces S_X and S_Y

16. More precisely, for each $s \in S$ and $i \in IVLS$, there exists a unique $t \in S$ such that *int*(*s, t*) = *i* (Lewin 1987, 26).
17. In determining the *Y*-layer, I will use what Lerdahl and Jackendoff (1983, 4) call "the final state of [the listener's] understanding"; this involves carefully reflecting on the rhythmic/metric perceptions that a given passage suggests, and then determining a final metric interpretation for the passage. "Final-state analysis" has not been without critics, as some consider it a static and one-dimensional approach to metric perception (Cady 1983; London 1997; Mirka 2009). However, as Temperley (2001, 14–19) has argued, this approach to meter can not only meaningfully model the unfolding listening experience, but also account for metrical revisions.
18. When I refer to "*X*-meter" and "*Y*-meter," the term "meter" simply refers to the number of pulses between

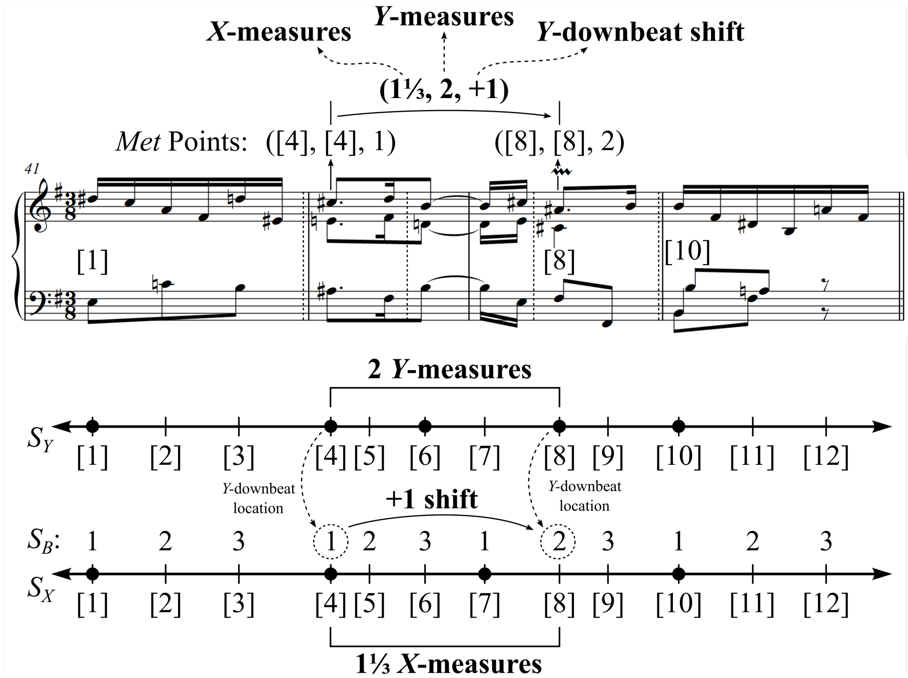


Figure 5. Summary of *Met*, using a simple hemiola from J. S. Bach's English Suite No. 5 in E minor, Gigue, mm. 41–44.

Audio Example 3. Hemiola in J. S. Bach's English Suite No. 5 in E minor, Gigue, performed by András Schiff (1990).

corresponding to the *X*- and *Y*-layers, respectively. In these spaces, bracketed integers correspond to eighth-note time points and black dots represent downbeats. A third, cyclic space, S_B, appears above S_X; this is the *Y*-downbeat shift space. Because the *X*-meter is 3, the possible *Y*-downbeat locations are beats 1, 2, 3, and all other rational beat locations b such that $1 \leq b < 4$. More simply, since the notated meter is 3/8, S_B is an idealized three-beat measure within which the heard downbeat may shift.[19]

The remaining annotations in Figure 5 illustrate how to measure the *Met* interval from the start of the hemiola at [4] to the F-sharp major chord at [8]. First, these musical events must be mapped into *Met* space. The chord initiating the hemiola occurs at time point [4] within S_X and S_Y, and the local *Y*-downbeat occurs on beat 1. Thus, this chord corresponds to the point

downbeats in a given layer. This is not to deny a richer metric hierarchy consisting of layers above and below the basic pulse layer, however, as discussed by Lerdahl and Jackendoff (1983), London (2004), and Mirka (2009), among others.

19. The components of *Met* shown in Figure 5 may suggest aspects of Inner Metric Analysis, which distinguishes between "inner" metric structures determined by note onsets and equidistant pulses and "outer" metric structures determined by time signatures and bar lines (Netske and Noll 2001; Volk 2008). While Inner Metric Analysis could certainly be used to justify a particular *Y*-metric reading, this form of analysis is not absolutely necessary for this paper's purposes. In particular, many of my *Y*-metric interpretations will depend on parameters beyond inner metric structure, such as melodic shape and phrase structure.

([4], [4], 1) in *Met* space. The chord at [8], on the other hand, maps to the point ([8], [8], 2), as it occurs at [8] in S_X and S_Y spaces and sits within a *Y*-measure whose downbeat is on beat 2 of the *X*-measure.[20] The cumulative *Met* interval, then, follows from prior observations: from [4] to [8], there are 1 1/3 *X*-measures (notated measures), 2 *Y*-measures (heard measures), and a *Y*-downbeat shift of +1 (from beat 1 to beat 2). Thus, the overall *Met* interval over the span in question is (1 1/3, 2, +1). It is important to note here that *Y*-downbeat shifts are calculated modulo the notated meter. In this case, the *Y*-downbeat shift is calculated mod 3 because of the notated 3/8 meter, so the +1 *Y*-downbeat shift could be rewritten as −2. Ultimately, the value chosen depends on the analyst's goals.

More formally, *Met* may be summarized as follows. First, *Met* is an example of a *direct product GIS*, which consists of multiple independent GISs fused to form a single, more complex GIS. Intervals in a direct product GIS are ordered *n*-tuples of intervals from the component GISs (Lewin 1987, 45). For instance, if *G* is a GIS containing the interval "4" and *H* contains the interval "−1," then the product GIS $G{\times}H$ must contain the interval (4, −1).

The GIS *Met* is a product of three independent GISs, one that counts bars in *X*-meter space, another that counts bars in *Y*-meter space, and a third that measures the *Y*-downbeat shift within S_B space (Wells 2017, par. 30). Points in *Met* space are ordered triples of the following form:

(S_X *time point*, S_Y *time point*, *Y-downbeat location within X-measure*).

In the spirit of Lewin, however, this study's primary concern will not be these points, but the intervals between the points, which take the following form:

(*#X-measures*, *#Y-measures*, *net Y-downbeat shift with respect to X*).

While the first two coordinates will be rational values, assuming one is measuring between rational time points, the third coordinate (though also rational) is calculated modulo the *X*-meter, as discussed above.[21] This value, when nonzero, will always be signed ("+2," "−1/2," and so forth). Finally, while the full, mathematical definition of *Met* implies independence of the component coordinates, the current study's applications of *Met* imply, in practice, a third coordinate that depends upon the first two.

The question of applying *Met* to Carnatic music must now be addressed. In Figure 6, I have transcribed a portion of the kriti *Jagadānandakāraka* by revered South Indian composer Tyāgarāja.[22] The transcribed excerpt may be heard in Audio Example 4, which is a pedagogical

20. Note that moving to time point [9] also yields a *Y*-downbeat on beat 2, as a single *Y*-measure can only have a single downbeat location—i.e., the corresponding *Met* point is ([9], [9], 2).
21. For the remainder of this article, I consistently assume a constant *X*-meter. While the construction of *Met* does permit multiple *X*-meters, the GIS is more complicated to use in these circumstances. For a discussion of these complications and necessary workarounds, see Wells (2017, pars. 27–29).
22. The *kriti* is a sophisticated form of Hindu devotional song consisting of three sections: the *pallavi*, the

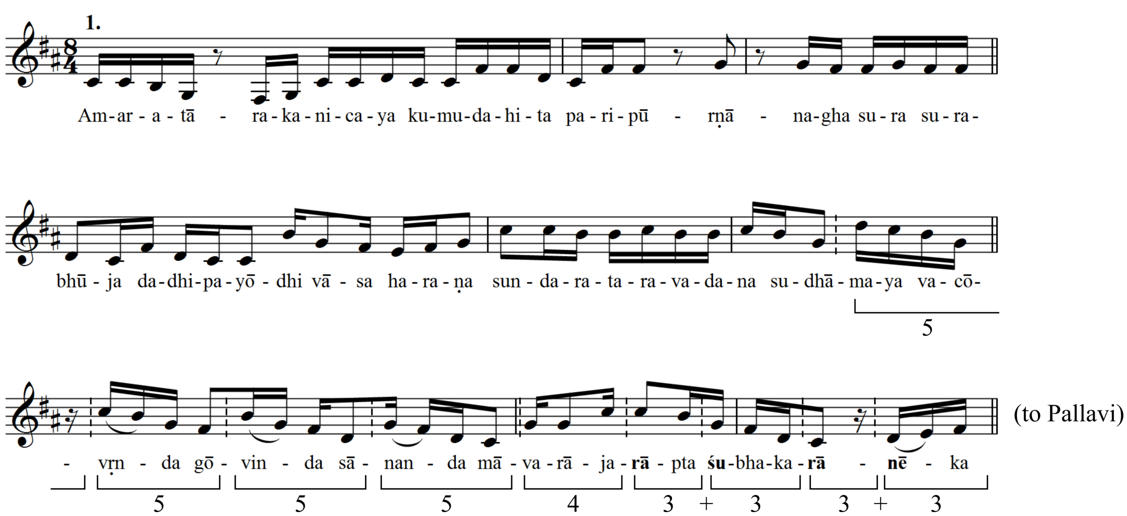


Figure 6. Tyāgarāja, *Jagadānandakāraka*, excerpt from *caraṇam* section, transcribed by the author.

Audio Example 4. Tyāgarāja, *Jagadānandakāraka*, first *caraṇam*, performed by Shivkumar Kalyanaraman.

recording by Shivkumar Kalyanaraman that I have chosen for its melodic clarity.

I represent the piece's Ādi tāḷa (4 + 2 + 2) using eight-beat measures subdivided into three angas each, where double solid bar lines mark out tāḷa cycles and single solid bar lines represent anga subdivisions. Observe, however, that near the end of the second cycle, a series of repeated melodic/rhythmic shapes lasting five "sixteenths" apiece initiates strong tension with the tāḷa. At the end of the excerpt appear several units of three, which are partly generated by long vowels in the text (in bold) and partly by the placement of longer rhythmic values. In between the fives and threes lies a mediating four-unit. The dotted bar lines interpret these melodic units as defining a *Y*-layer that conflicts with the eight-beat tāḷa (*X*-layer). It is worth noting that the *Y*-layer over the conflicting region demonstrates the traditional *gopucca* or "cow-tail" rhythmic shape, consisting of progressively shorter rhythmic units.[23]

The tension with the tāḷa in the aforementioned region can be further quantified using *Met* intervals, as shown in Figure 7. The figure illustrates how to measure the *Met* interval

anupallavi, and one or more *caraṇams*. Traditionally, the pallavi returns in abbreviated form after the anupallavi and each caranam, thus serving as a sort of refrain. Figure 6 presents the first of ten caraṇams in *Jagadānandakāraka*. For more on the kriti and its significance in Carnatic music, see Viswanathan and Allen (2004, 15–29).

23. Nelson (2008, 19) points out that the gopucca shape is "perhaps the most widely used design type in Karnatak rhythm." Other standard rhythmic shapes (*yati*) include *srotovaha* ("river mouth"), *damaru* ("hourglass"), *mridanga* (named for the *mridangam* drum), *sama* (uniform shape), and *visama* (random shape); see Nelson (2000, 147). Subramaniam and Subramaniam (1995, 73–74) use the same terms to classify tāḷas according to the sizes and distributions of their angas.

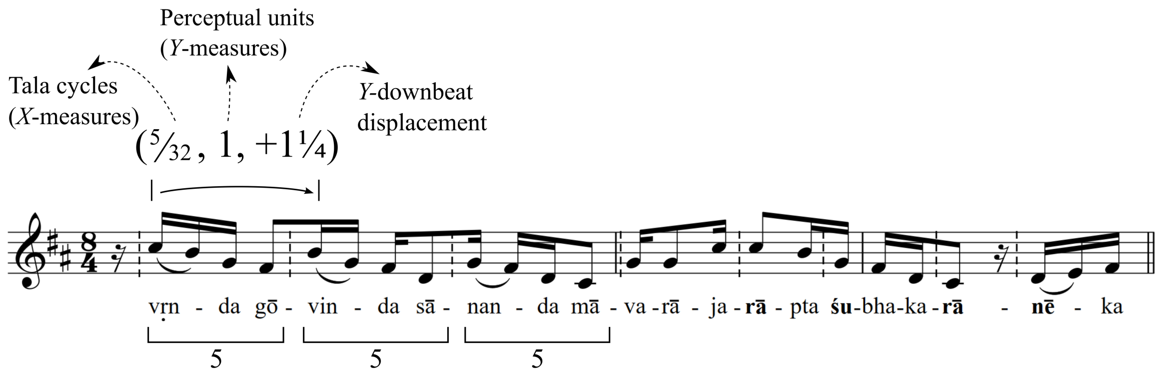


Figure 7. Calculating *Met* intervals in Carnatic music.

across one of the 5-units. Namely, given eight-beat tāḷa cycles, where the quarter-note represents the basic pulse, the bounded melodic unit spans 5/32 of a cycle (*X*-measure) and one *Y*-measure, with an overall *Y*-downbeat displacement of +1 1/4. Thus, this 5-unit (like those that follow) spans the *Met* interval (5/32, 1, +1 1/4).

Figure 8 fills in the remaining intervals over the latter portion of the Tyāgarāja excerpt. Starting with the 5-units, the music generates a series of four (5/32, 1, +1 1/4) intervals, a mediating (1/8, 1, +1) interval, and four (3/32, 1, +3/4) intervals leading into the pallavi return. Now, observe that every interval I have highlighted spans exactly one *Y*-measure. By comparison, one could also measure across the entire conflicting region to yield the cumulative interval (1 1/8, 9, +1). Breaking down a larger interval into subintervals that span single *Y*-measures is often analytically useful, as it reveals the larger interval's underlying components, much like the prime factorization of a positive integer. This special intervallic decomposition, in which an interval

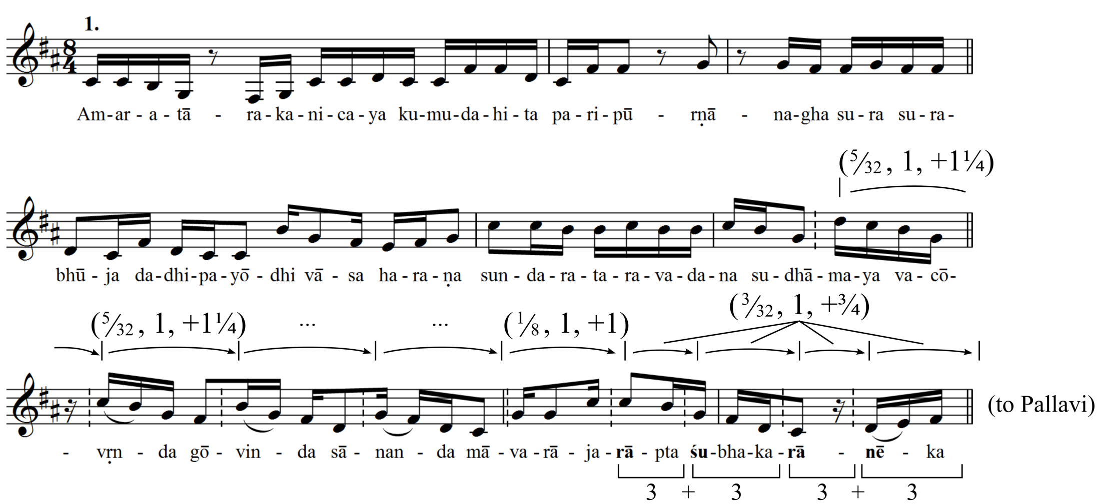


Figure 8. *Met* intervals in Tyāgarāja's *Jagadānandakāraka*.

is broken down at *Y*-downbeats, I call the *Y-decomposition* of the larger interval.[24] Numerically, the *Y*-decomposition of the interval spanning the metrically conflicting passage in question is

$$(1\ 1/8, 9, +1) = 4(5/32, 1, +1\ 1/4) + (1/8, 1, +1) + 4(3/32, 1, +3/4),$$

where the cumulative interval appears on the left and the subintervals appear on the right. One could similarly define decompositions that subdivide intervals at *X*-downbeats or at both *X*- and *Y*-downbeats, although these decompositions will not be of interest for the current project. More significant will be a decomposition unique to Carnatic music, the anga decomposition, which I define in a later section.

The final theoretical building block for the current study is how to use *Met* to model trikāla technique. As such, consider a simple exercise adapted from Viswanathan and Allen (2004, 39) that I have transcribed and annotated in Figure 9. This trikāla exercise consists of a solkaṭṭu (rhythmic solfège) pattern that is performed at progressively faster speeds against constant Ādi tāḷa. Under the assumption that each statement of the solkaṭṭu pattern defines one *Y*-measure, the *Met* intervals transform each time the speed increases, as shown in the Figure 9.

In *Met* terminology, a series of "intervallic contractions" occur from one āvarta (tāḷa cycle) to the next. Specifically, *intervallic expansion and contraction* involve scaling a span of *Y*-measures by some positive rational value *k* over a constant *X*-meter.[25] Intervals formed at different levels of metric hierarchy may thus be viewed as transformations of one another. Figure 10 illustrates intervallic contractions and expansions generated by standard melodic

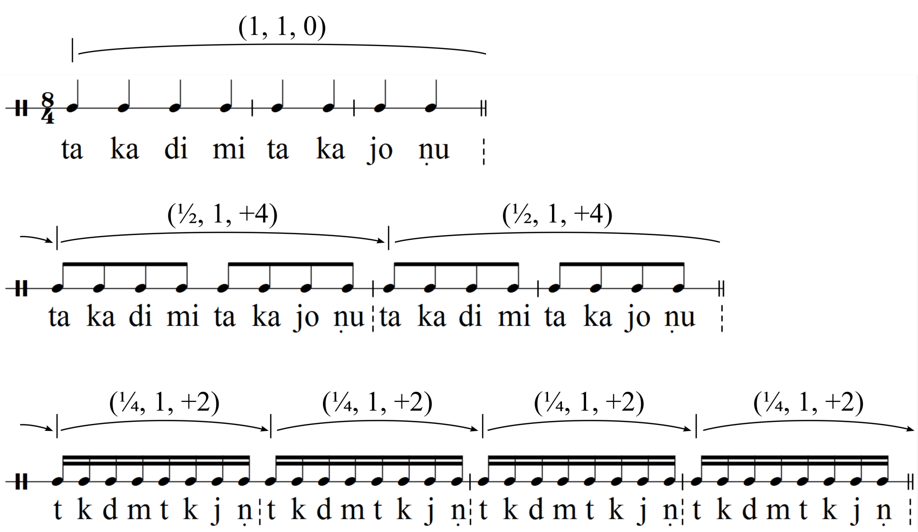


Figure 9. Trikāla technique in a simple Ādi tāḷa solkaṭṭu exercise, adapted from Viswanathan and Allen (2004, 39).

24. For the formal definition of the *Y*-decomposition, see Wells (2015a, 90–94).
25. For a full, mathematically formal theory of intervallic expansions and contractions, see Wells (2015a, 103–34).

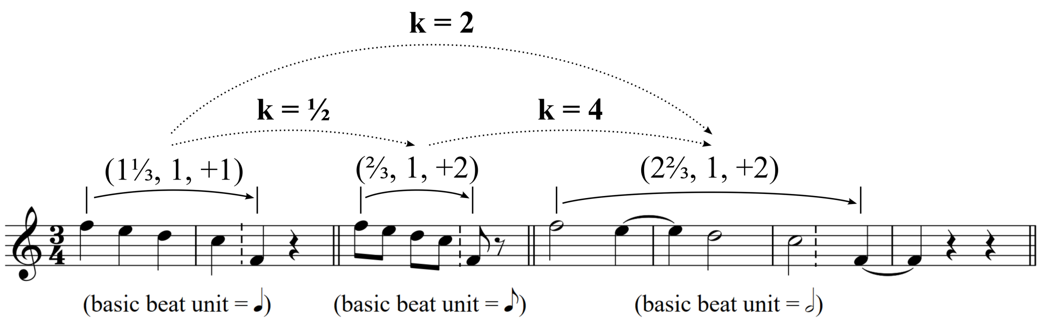


Figure 10. Intervallic contractions/expansions generated by diminution and augmentation of a melody.

diminution and augmentation. Here a simple melody is transformed to appear at various levels of pulse over a constant 3/4 meter. Notice, however, that the changing *Met* intervals in the diagram are responding not to specific note values or melodic content, but to the changing *Y*-pulse unit; thus, intervallic expansion and contraction are more generalized operations than traditional augmentation or diminution.

At this point, one might wonder if *Met* intervals expand and contract with any sort of numerical predictability. In fact, the following two propositions, proven in Wells (2015a), precisely predict the numerical outcome of any rational expansion/contraction operation:

> **Proposition 1** (Wells 2015a, 117; 2017, par. 43): For some x, $y \geq 0$, suppose (x, y, a) is a *Met* interval measured between two *Y*-downbeats, and assume the *X*-meter has constant value m. Then for any positive rational k, the k-expansion/contraction of (x, y, a) is given by
>
> $$(kx, y, kxm).$$
>
> **Proposition 2** (Wells 2015a, 117–18; 2017, par. 44): Suppose (x, y, a) is a *Met* interval where x, $y \geq 0$, and assume the *X*-meter has constant value m. Then for any positive integer k, the k-expansion of (x, y, a) is given by
>
> $$(kx, y, ka).$$

Note that while Proposition 1 requires that the starting interval be measured between *Y*-downbeats, Proposition 2 does not. On the other hand, while Proposition 2 only applies to positive integer expansions, Proposition 1 allows k to take on any positive rational value. Additionally, it is essential to remember that the values in the third coordinate (*Y*-downbeat shift) are calculated mod m.

For example, using these propositions, one could calculate the numerical 1/2-contraction of the first (1, 1, 0) interval in the above Ādi tāḷa exercise (Figure 9). Given that the expansion value ($k = 1/2$) is not an integer, Proposition 2 does not apply. Because the interval (1, 1, 0) spans two *Y*-downbeats and $k = 1/2$ is rational, however, Proposition 1 applies. The exercise's constant *X*-meter of 8 means that $m = 8$; thus, the interval $(x, y, a) = (1, 1, 0)$ contracts to $(kx, y, kxm) = (1/2 \cdot$

1, 1, 1/2 · 1 · 8) = (1/2, 1, +4). Observe that this result matches the interval values appearing in the second tāḷa cycle of Figure 9.

Figure 11 fills in the remaining intervallic contractions in the trikāla exercise: there are two consecutive 1/2-contractions and, from the first āvarta to the third, a 1/4-contraction. Additionally, Figure 11 indicates the cumulative interval over each tāḷa cycle (boldface intervals to the right). Observe that despite internal *Y*-downbeat shifting in the second and third cycles, the *Y*-downbeat shift for each cycle as a whole is zero. Additionally, while the *X*-coordinate of each interval is a constant 1, the *Y*-coordinates are successively doubled. As such, these intervals depict a balance between stability and change that undergirds the rhythmic exercise. As the next two sections will show, however, rhythmic/metric stability and change can operate at much higher levels of complexity than what appears in this simple exercise.

Analysis: Purandara Dāsa's *Alankārams*

Having established this study's central theoretical framework, I now turn to an important set of traditional exercises that introduce budding students of Carnatic music to trikāla technique in various tāḷas. These five-hundred-year-old exercises, called *alankārams*, are part of a progressive curriculum devised by Purandara Dāsa, generally considered the "father" of Carnatic music. Typically, students progress through four major groups of exercises in rāga

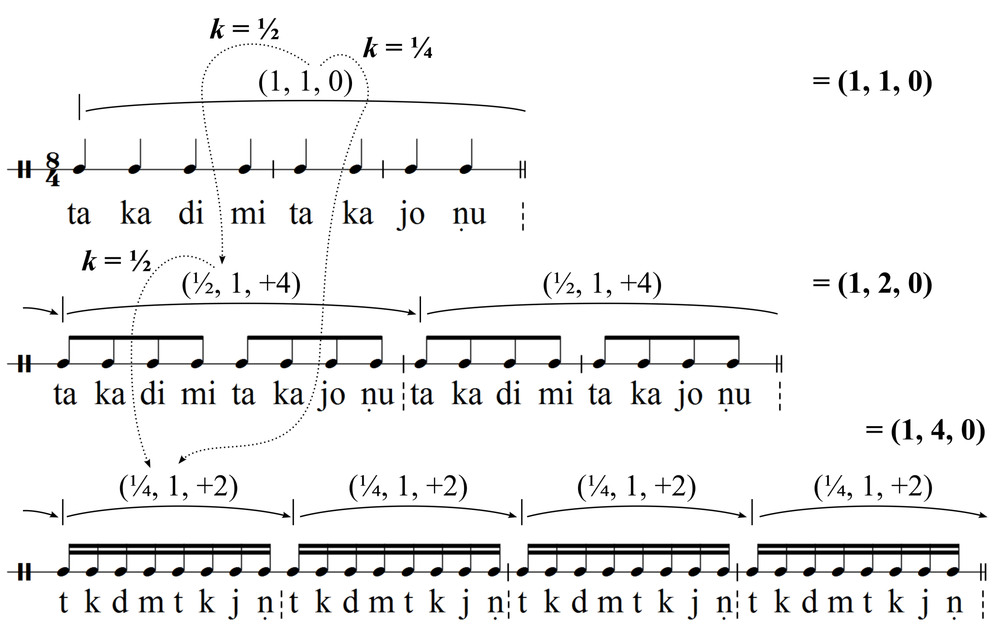


Figure 11. Trikāla technique yielding intervallic contractions, after exercise from Viswanathan and Allen (2004, 39).

Māyāmāḷavagowḷa,[26] with each exercise group emphasizing a different technical challenge, before attempting the alankārams.[27] Students are expected to learn all exercises, including the alankārams, in three to four speeds—that is, using trikāla technique. Unlike the initial exercises, which are all in Ādi tāḷa, the alankārams require proficiency in all seven of the main classifications of tāḷa, creating new, more challenging contexts for trikāla.[28] Figure 12 illustrates the alankāram for Tiṣra Jāti Tripuṭa tāḷa (seven beats partitioned as 3 + 2 + 2). The kriyās are "clap, pinky, ring, clap, wave, clap, wave."[29] Below each note is a corresponding *sargam* (melodic solfège) syllable, where I have arbitrarily chosen the note C to represent "sa," the first note of the rāga.[30]

To complete the exercise in "first speed" (one syllable per beat), as shown in Figure 12, the student sings the opening seven-note pattern from "sa," "ri," "ga," "ma," and then "pa." An inversion of the original pattern is then sung starting on high "sa," "ni," "dha," "pa," and "ma," with the final iteration ending back on the original "sa." From this point, the student repeats the exercise in second speed (two syllables per beat), third speed (four syllables per beat), and possibly fourth speed (eight syllables per beat), each of which introduces a new form of tension with the underlying tāḷa. In Figures 13 and 14, I present transcriptions of the complete second- and third-speed versions of the exercise. Observe that in third speed, in order for the performer to finish at the end of a tāḷa cycle, the entire alankāram must be performed twice.

Part of the alankāram's complexity in second and third speeds is that the student must manage three incommensurate values at once: division of the beat into two or four equal parts; a seven-beat tāḷa; and a five-fold repetition of the initial seven-syllable pattern in each half of the alankāram. To unravel how these tensions play out over the course of the exercise, and depict how the student's performative experience evolves, I now consider the *Met* intervals at play. Figure 15 provides a *Met*-intervallic analysis of the opening seven-syllable pattern in first, second, and third speeds.

---

26. Māyāmālavagowla resembles the "double harmonic" scale—namely, a major scale with the second and sixth notes flatted.
27. The *saraḷi variṣai* provide an introduction to simultaneously working with melody, rhythm, and tāḷa; the *jaṇṭai variṣai* teach proper ornamentation of repeated notes; the *mēlsthāyi variṣai* expand the student's command of different registers; and the *dāṭu variṣai* provide practice with melodic leaps within the rāga. The entire set of exercises, in traditional notation, can be found in Ravikiran (2012).
28. The traditional tāḷa scheme (*sūḷādi sapta tāḷa*) consists of seven tāḷa types, each of which contains five individual tāḷas, creating a 35-tāḷa scheme overall. In addition to the new metric challenges the alankārams represent, these exercises mark the pedagogical stage at which students begin learning *gamaka*s, characteristic vocal ornaments that help define each rāga (see Pesch 1999, 80–81).
29. This tāḷa is in the same family (Tripuṭa) as Ādi tāḷa; note that the only difference between the two is the length of the first anga (three vs. four beats). The "Tiṣra Jāti" descriptor means that any angas consisting of a clap followed by finger counts (*laghu*) must be three beats in length. Ādi tāḷa, whose laghu is four beats long, is more formally known as "Chaturaṣra Jāti Tripuṭa tāḷa."
30. The Indian sargam sequence "sa, ri, ga, ma, pa, dha, ni, sa" is roughly analogous to "do, re, mi, fa, sol, la, ti, do" in Western movable-do solfège. In the South Indian system, however, the intervals between syllables vary depending on the rāga. For instance, "ri" could occur in a natural form, a flat form (as in rāga Māyāmālavagowla), or a sharp form.

Figure 12. Alankāram in Tiṣra Jāti Tripuṭa tāḷa, first speed.

Figure 13. Alankaram in Tiṣra Jāti Tripuṭa tāḷa, second speed.

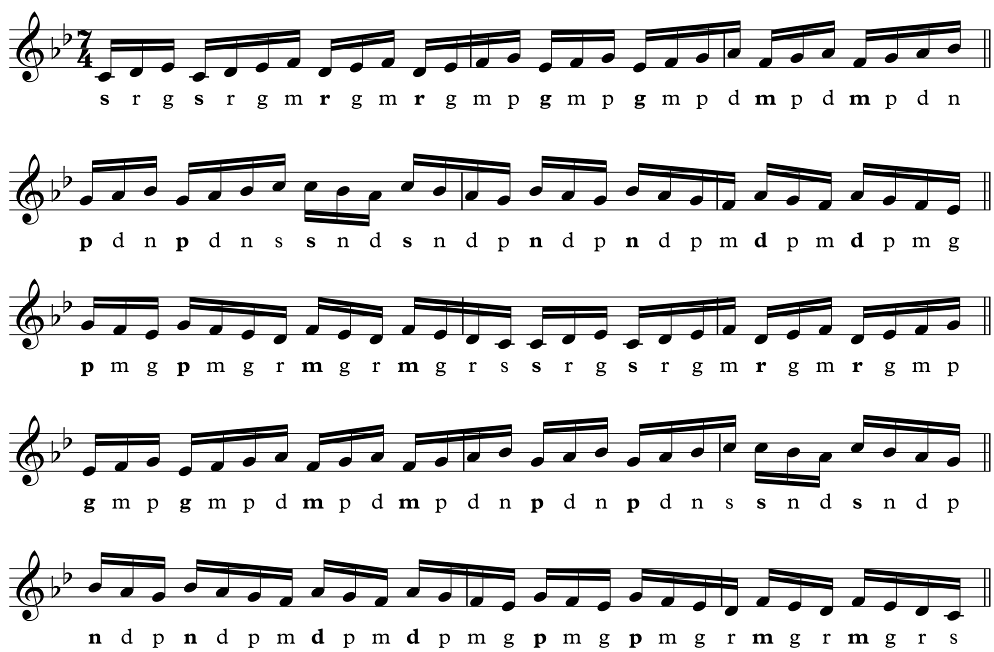


Figure 14. Alankāram in Tiṣra Jāti Tripuṭa tāḷa, third speed.

In Figure 15, I have broken each seven-syllable pattern—henceforth, the "Basic Pattern"—into a pair of phrases comprising three and four sargam syllables, respectively. For convenience, I designate the three-syllable phrase "Phrase A" and the four-syllable phrase "Phrase B." I have marked phrase beginnings with bold syllables and dotted bar lines, as well as through phrase-based beaming in the second and third speeds. The uppermost horizontal

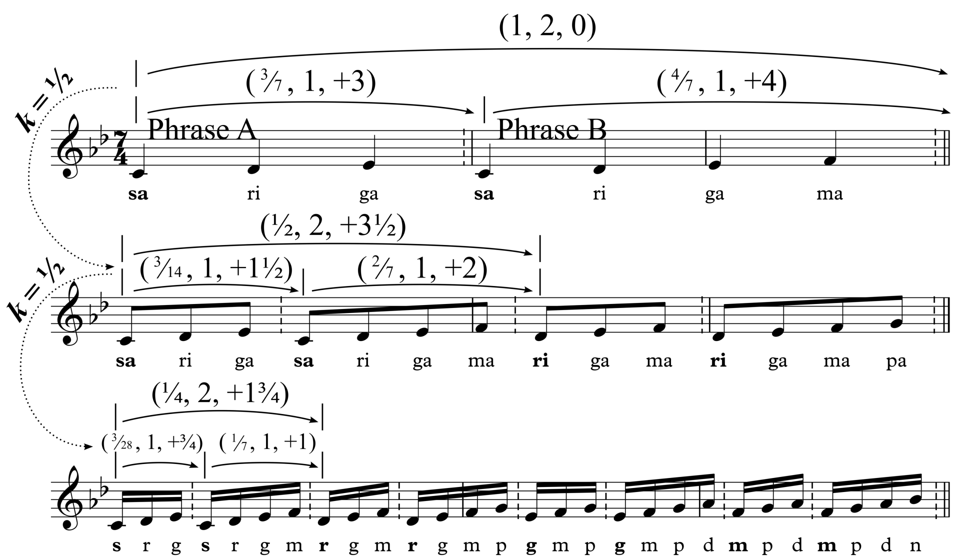


Figure 15. *Met*-intervallic analysis of the Tripuṭa tāḷa alankāram's opening melodic pattern in first, second, and third speeds.

arrow in each speed indicates the *Met* interval across the entire, two-phrase Basic Pattern. Thus, for instance, the Pattern spans the interval $P = (1, 2, 0)$ in first speed, but spans $(1/2, 2, +3\ 1/2)$ in second speed and $(1/4, 2, +1\ 3/4)$ in third speed. I designate the latter two intervals P_2 and P_3, respectively. Like in Figure 11, each speed change induces an intervallic 1/2-contraction. As a result, while each Pattern continues to span two phrases (Y-measures), the proportion of the tāḷa spanned (X-measures) and resulting Y-downbeat shift values diminish.

More interesting, perhaps, are the Y-decompositions of these intervals, indicated in Figure 15 by arrows spanning the individual phrases. While the phrase-spanning intervals' X-coordinates demonstrate a proportional shrinking within the seven-beat tāḷa as the speed increases, the Y-downbeat shift values represent how the phrase beginnings migrate through the tāḷa as the student performs the exercise. For instance, the Y-decomposition of P_2 (the second-speed Basic Pattern interval) is

$$(1/2, 2, +3\ 1/2) = (3/14, 1, +1\ 1/2) + (2/7, 1, +2),$$

or

$$P_2 = A_2 + B_2$$

where A_2 and B_2 are the intervals spanning second-speed Phrases A and B, respectively. Thus, one iteration of P_2 shifts the Y-downbeat forward by 3 1/2 beats mod 7 (half of a tāḷa cycle), and this larger shift can be decomposed into smaller shifts by 1 1/2 beats (via A_2) and 2 beats (via B_2), respectively.

The +1 1/2 downbeat shift of A_2 implies a different rhythmic/metric function from the first-speed intervals $A = (3/7, 1, +3)$ and $B = (4/7, 1, +4)$, each of which contains an integer-valued Y-downbeat shift. Specifically, in second speed, Phrase A (via A_2) continually moves the performer between "on-the-beat" and "between-the-beats" states.[31] On the other hand, B_2, with its integer-valued shift (+2), represents stasis: it can only maintain the "on-the-beat" or "between-the-beats" state generated by A_2. For example, in the first statement of the second-speed Basic Pattern, A_2 shifts the Y-downbeat to a "between-the-beats" state that B_2 then maintains, since Phrase B begins and ends between beats. When the Pattern is repeated starting on "ri," A_2 then shifts the Y-downbeat back into alignment with the main beat; B_2 maintains this alignment.[32]

In third speed, the Y-decomposition introduces new fractional values:

$$(1/4, 2, +1\ 3/4) = (3/28, 1, +3/4) + (1/7, 1, +1),$$

or

31. The beat state transformations generated by A_2 suggest Ng's (2005; 2006) "hemiolic cycle," an extended hemiola structure Ng locates in several Brahms works. While the hemiolic cycle requires triple meter and three discrete beat states, *Met*-based transformations can be defined for any meter and for all rational-valued Y-downbeat shifts.
32. I encourage the reader to sing or speak the second-speed line in Figure 15 while performing the kriyās ("clap, pinky, ring, clap, wave, clap, wave") on each "quarter-note" beat to experience the shift and stasis to which I am alluding.

$$P_3 = A_3 + B_3.$$

The X-coordinate values are, unsurprisingly, rather small at this point, representing the Basic Pattern's notable shrinking against the unchanging backdrop of Tripuṭa tāḷa. It is still the third coordinate, though—the Y-downbeat shift—that bears the most performative significance. Although Phrase A again transforms the music's relationship to the beat while Phrase B maintains stasis, the nature of Phrase A's change is different: rather than +1/2, the Y-downbeat shift within A_3 is +3/4. Two important points follow: (1) in relation to the beat unit (akṣara), A_3 induces a progressive shift to the left by a quarter of a beat; and (2) four iterations of A_3 are necessary to return to an "on-the-beat" state. To clarify, starting from the beginning of the third-speed exercise, one iteration of A_3 (with its +3/4 shift) results in a Y-downbeat just to the left of the nearest beat; two iterations of A_3 mean 2(+3/4) = +1 1/2, so that the new Y-downbeat is midway between two beats; three iterations imply 3(+3/4) = +2 1/4, resulting in a Y-downbeat just to the right of the nearest beat; and four iterations mean 4(+3/4) = +3, so that the "on-the-beat" state is restored. Bear in mind that through all of these repetitions, B_3 (with its +1 shift) simply maintains whatever state A_3 has generated with respect to the beat.

To make matters more complex, recall that the Basic Pattern is stated five times ascending and five times descending before the original "sa" is achieved. Thus, a four-fold repetition of the Pattern in third speed ($4P_3$) may restore an "on-the-beat" state, but it does not complete the cumulative melodic ascent. At this point, it may be useful to zoom out and again consider the intervals above the uppermost arrows in Figure 15. Iterating each of P, P_2, and P_3 five times yields the following results (recall that the third coordinate is calculated mod 7):

First Speed: $5P = 5(1, 2, 0) = \mathbf{(5, 10, 0)}$

Second Speed: $5P_2 = 5(1/2, 2, +3\ 1/2) = (2\ 1/2, 10, +17\ 1/2) = \mathbf{(2\ 1/2, 10, +3\ 1/2)}$

Third Speed: $5P_3 = 5(1/4, 2, +1\ 3/4) = (1\ 1/4, 10, +8\ 3/4) = \mathbf{(1\ 1/4, 10, +1\ 3/4)}$

While each resulting boldface interval spans ten sung phrases (represented by the middle coordinate), the number of tāḷa cycles spanned is, with each speed increase, halved (X-coordinate). The Y-downbeat shift values change in response, with $5P_3$ enacting a net Y-downbeat shift of +1 3/4. Hence, after five third-speed Basic Pattern iterations, the performer will finish just short of beat three of the tāḷa (beat 1 + 1 3/4 Y-downbeat shift = beat 2 3/4). While five iterations of P_3 complete the Basic Pattern's ascent, they force the Y-downbeat cycle generated by A_3 to begin anew. Similarly, in second speed, four iterations of P_2 result in an "on-the-beat" state, while the fifth iteration, which completes the Basic Pattern's ascent, reactivates a "between-the-beats" state.

Obviously, if the exercise is continued long enough, realignment with the tāḷa will eventually occur. In second speed, realignment occurs at the end of the exercise, after the five inverted Basic Pattern statements have occurred. In third speed, however, the entire exercise (five ascending Basic Patterns and five descending, inverted Basic Patterns = $10P_3$) must be performed twice for realignment to occur, for $10P_3$ yields a +3 ½ Y-downbeat shift, while $20P_3$ yields a net +7 = 0 shift. Ultimately, then, the boldface intervals above ($5P_2$ and $5P_3$) must be doubled and quadrupled, respectively, yielding (5, 20, 0) for the entire second-speed performance and (5, 40, 0) for third speed.

More important, though, are the subtle tensions between melody and tāḷa that materialize, transform, and disappear over the course of the alankāram, with A_2 and A_3 generating new metric states and B_2 and B_3 prolonging these states. The *Met* intervals numerically represent the Basic Pattern's evolving relationship with the tāḷa throughout the exercise, with each Pattern iteration representing a new metric experience for the performer. These intervals also have predictive power: for instance, one could easily calculate the metric implications of performing three iterations of the Basic Pattern (or its inversion) in second speed, thirteen iterations in third speed, or even fifty iterations in fourth speed.[33]

The methods provided here need not be limited to Tripuṭa tāḷa, however. Figure 16 presents another alankāram, this one in Chaturaṣra Jāti Dhruva tāḷa, which comprises fourteen beats partitioned as 4 + 2 + 4 + 4.[34] The figure illustrates a new Basic Pattern, which I

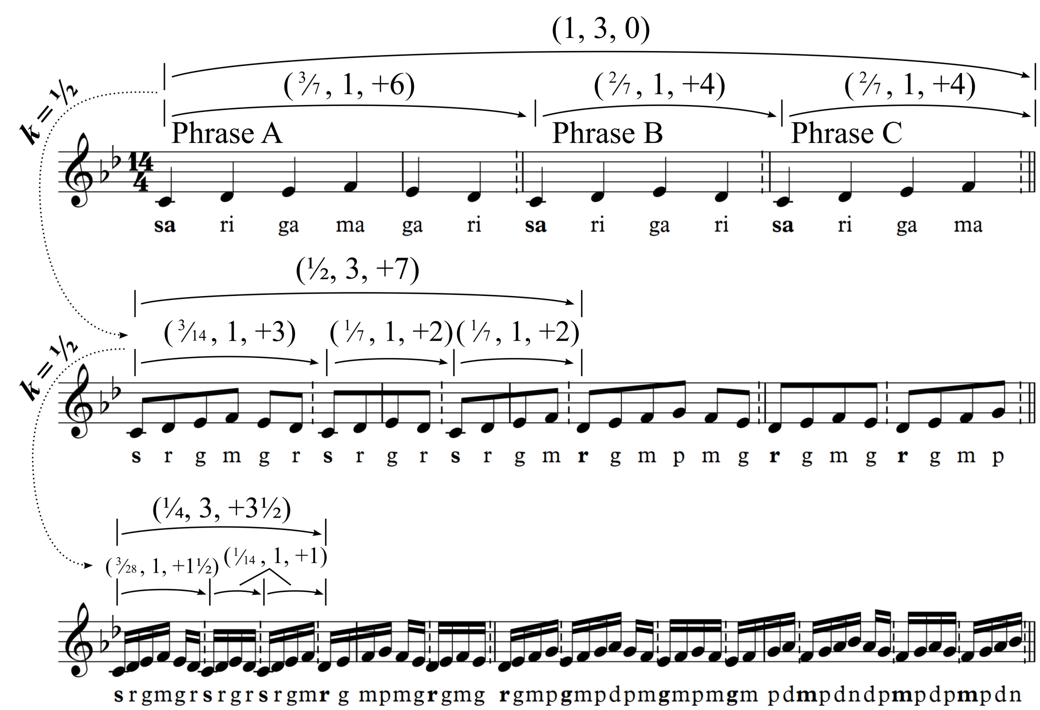


Figure 16. *Met*-intervallic analysis of the Dhruva tāḷa alankāram's opening melodic pattern in first, second, and third speeds.

33. Fifty iterations in fourth speed, for instance, would yield $50P_4 = 50(1, 2, 0)_{k=1/8} = 50(1/8, 2, +7/8) = (6\ 1/4, 100, +1\ 3/4)$. Note that at this speed, one hundred phrases only require 6 1/4 tāḷa cycles.
34. The kriyās for this tāḷa are "clap, pinky, ring, middle, clap, wave, clap, pinky, ring, middle, clap, pinky, ring, middle."

have partitioned into Phrases A, B, and C. Note that in first speed, Phrase A spans two angas, while Phrases B and C span one each.

When these phrases are successively contracted via trikāla technique, the metric implications are even more complex than in the Tripuṭa tāḷa alankāram, as the melody and tāḷa are longer and contain more subdivisions. In second speed, one iteration of the Basic Pattern spans the interval

$$(1/2, 3, +7) = (3/14, 1, +3) + (1/7, 1, +2) + (1/7, 1, +2),$$
or
$$P_2 = A_2 + B_2 + C_2.$$

The composite interval P_2 shows that in second speed, the three-phrase Basic Pattern lasts half of a tāḷa cycle and moves the Y-downbeat forward seven beats within the fourteen-beat cycle. The values of A_2, B_2, and C_2 indicate that Phrases B and C have mutually comparable metrical effects (since $B_2 = C_2$), while Phrase A, being longer, has differing implications. In fact, the intervals $A_2 = (3/14, 1, +3)$ and $B_2 = C_2 = (1/7, 1, +2)$ provide a key to understanding how the metric function of Phrase A differs from that of B and C.

First, the interval A_2, with its +3 Y-downbeat shift, has the power to enact a *parity shift*, moving the Y-downbeat from an even beat to an odd beat within the tāḷa, or from an odd beat to an even beat.[35] Consider, for instance, the first iteration of A_2 in Figure 16 (middle system, first six notes). This interval shifts the Y-downbeat from beat 1, an odd beat, to beat 4, an even beat. B_2 and C_2, on the other hand, with their +2 Y-downbeat shifts, maintain the given parity, moving the Y-downbeat to beats 6 and 8, respectively. From this point, A_2 enacts another Y-downbeat shift to restore odd parity, which B_2 and C_2 then maintain.

Notions of parity shift (A_2) and parity preservation (B_2 and C_2) have special significance in the current tāḷa. Namely, unlike in Tripuṭa tāḷa, the anga-initiating claps in Dhruva tāḷa consistently fall on odd beats (1, 5, 7, and 11). Thus, when A_2 shifts the Y-downbeat from odd to even parity, the subsequent B and C phrases become skewed with respect to the odd-beat claps, expressing a new tension with the tāḷa. This tension is only resolved when Phrase A reappears, restoring odd parity to the Y-downbeat via A_2.

The third-speed version of this alankāram introduces new roles for Phrases A, B, and C, as suggested by the intervallic decomposition

$$(1/4, 3, +3\ 1/2) = (3/28, 1, +1\ 1/2) + (1/14, 1, +1) + (1/14, 1, +1),$$
or
$$P_3 = A_3 + B_3 + C_3,$$

spanning one iteration of the Basic Pattern. While A_3, B_3, and C_3 span the same number of Y-

35. My notion of "parity shift" recalls Ng's (2005, 183) notion of the "duple opposition," which is the perceived reversal of strong and weak beats in duple meter.

measures as their first- and second-speed counterparts (1 Y-measure apiece, for a total of 3 Y-measures within P_3), the X-measure and Y-downbeat shift values significantly differ. In particular, the X-coordinate of P_3 shows that the Basic Pattern now spans a quarter of a tāḷa cycle, generating an overall Y-downbeat shift by +3 1/2 (mod 14). Now, recall that because of A_2, each statement of $P_2 = (1/2, 3, +7)$, with its +7 net Y-downbeat shift, resulted in a Y-downbeat parity shift. Each P_3 statement, however, moves the Y-downbeat between "on-the-beat" and "between-the-beats" states, much like what occurred in the second-speed Tripuṭa tāḷa alankāram.

The A_3, B_3, and C_3 intervals indicate the origins of these new states. As in the second-speed Tripuṭa tāḷa alankāram, the Phrase A interval (A_3), with its +1 1/2 Y-downbeat shift, is the musical agent that moves the Y-downbeat onto or off of the beat. B_3 and C_3, with their +1 Y-downbeat shifts, maintain the on- or off-the-beat state generated by A_3, much like B_2 in the Tripuṭa tāḷa alankāram. Now, observe that in third-speed Dhruva, two Basic Pattern iterations generate the new interval

$$2P_3 = 2(1/4, 3, +3\ 1/2) = (1/2, 6, +7).$$

Thus, six phrases (two Basic Patterns) in third-speed Dhruva span half of the tāḷa cycle and generate a 7-beat Y-downbeat shift—another parity shift. Two more Basic Patterns will, then, fill out the tāḷa cycle and restore the original parity, yielding the interval

$$4P_3 = 4(1/4, 3, +3\ 1/2) = (1, 12, 0).$$

As in Tripuṭa tāḷa, though, the alankāram's overall structure consists of five ascending Basic Patterns followed by five descending Basic Patterns, the last of which ends on the low "sa" on which the exercise began. Thus, the entire third-speed alankāram (with no repetitions) generates the interval

$$10P_3 = 10(1/4, 3, +3\ 1/2) = (2\ 1/2, 30, +35) = (2\ 1/2, 30, +7).$$

The resulting Y-downbeat, then, is in the middle of a tāḷa cycle. Because the exercise cannot end in the middle of the tāḷa, it must be iterated until the Y-downbeat returns to beat 1. Thus, since the whole exercise in third speed ($10P_3$) yields the interval (2 1/2, 30, +7), two performances yield 2(2 1/2, 30, +7) = (5, 60, 0). This is the minimum number of iterations needed to restore Y-downbeat alignment, spanning five full tāḷa cycles and 60 phrases, or 20 Basic Patterns.

In sum, these analyses of the Tripuṭa and Dhruva tāḷa alankārams suggest several ways in which *Met* intervals can model trikāla technique. First, the evolving relationships between the tāḷa and the melodic/rhythmic phrasing are encapsulated in the three *Met* interval coordinates. In particular, the Y-downbeat shift values imply differing metric roles for the phrases making up the Basic Pattern. More importantly, they depict the evolving challenges experienced by the singer, who must manage several simultaneous metric strands whose mutual relationships are in flux. As such, in representing the shifting metric states (Y-

downbeat parity, on- vs. off-beat states, and so forth), the *Met* intervals model a crucial component of the musical experience.

Additionally, the GIS-based mathematics allows one to predict the implications of iterating a melody at multiple speeds. While the previous examples only considered first, second, and third speeds, one could easily calculate *Met* intervals for faster speeds such as fourth and fifth speeds. On the one hand, these calculations predict local interactions between melodic phrasing and tāḷa, allowing precise analysis of phrases' varied metric roles. On the other hand, the calculations can reveal aspects of global melodic structure, such as the number of pattern iterations necessary to restore alignment with the downbeat (sam) at a given speed. The power of the *Met* intervals, in these situations, results from their capability of measuring linear time (number of tāḷa cycles and melodic units spanned) and modular time (position within a tāḷa cycle) simultaneously.[36] Additionally, the intervals reify specific forms of metric tension so that they may form part of an analytical narrative. Overall, the mathematics can provide a useful model of the moment-to-moment experience of performing the alankārams in multiple speeds, while also revealing the implications of trikāla technique on a broader scale.

Analysis: RTP Performance Revisited

Having examined the rhythmic/metric subtleties of the pedagogical alankāram exercises, I now return to the virtuosic RTP performance that opened this study (Figure 3) and attempt to gain deeper insights into the trikāla section using *Met*. Recall that the basic pallavi melody consists of A and B phrases, and the underlying tāḷa is Miṣra Cāpu (3 + 2 + 2). Figure 17 presents a *Met*-intervallic interpretation of the first full A–B melodic statement. Note that the overall intervallic span of the melody, *Y*-decomposed, is

$$(4, 6, 0) = 4(1/2, 1, +3\ 1/2) + 2(1, 1, 0),$$

where the $4(1/2, 1, +3\ 1/2)$ portion corresponds to the A phrase and the $2(1, 1, 0)$ portion to the B phrase. The phrases' defining intervals—$(1/2, 1, +3\ 1/2)$ and $(1, 1, 0)$, respectively—reflect their differing relationships to the tāḷa.

As to the nature of these relationships, Widdess (1977, 64) convincingly argues that A and B are in "two different tempi" against the constant tāḷa, with A "twice as fast" as B. The above *Met* intervals suggest a way to clarify this observation. Namely, the basic $(1/2, 1, +3\ 1/2)$ and $(1, 1, 0)$ intervals defining A and B, respectively, are related by a simple 2-expansion, as an application of Proposition 2 reveals:

$$(1/2, 1, +3\ 1/2)_{k=2} \longrightarrow (2 \cdot 1/2, 1, 2 \cdot +3\ 1/2) = (1, 1, +7) = (1, 1, 0),$$

36. My conceptions of linear and modular time here correspond roughly to Morris's (1987) "measured time," or "m-time" (299), and "modular time of order *n*" or "mod-time" (301–302).

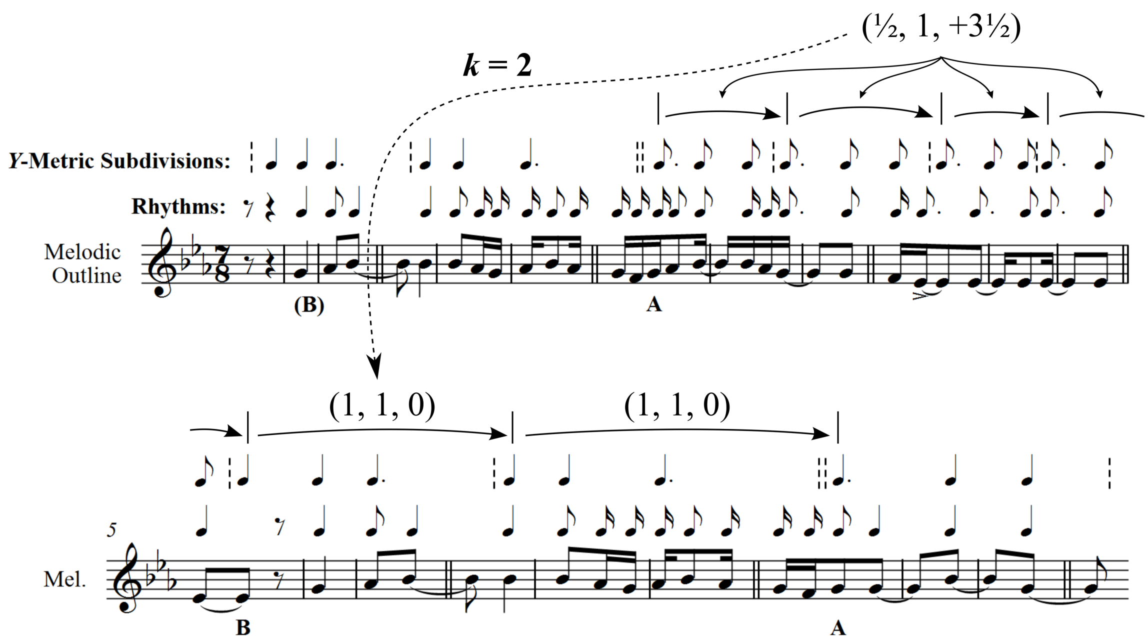


Figure 17. *Met*-intervallic span of basic RTP melody: $(4, 6, 0) = 4(1/2, 1, +3\ 1/2) + 2(1, 1, 0)$.

where the last equality holds because the *X*-meter (tāḷa) has value 7, and $+7 \equiv 0 \pmod 7$. The transformed *Y*-downbeat shift value ($+3\ 1/2 \rightarrow 0$) shows that this 2-expansion locally stabilizes the *Y*-downbeat, resulting in a constant *Y*-downbeat location of 2 throughout the B portion of the melody.

I represent this 2-expansion in Figure 17 by a dashed arrow connecting the basic A and B intervals. Observe that the entirety of B is *not* a perfect 2-expansion of A, however, as a full 2-expansion of A would generate $4(1, 1, 0)$ instead of $2(1, 1, 0)$. Thus, while B certainly responds to and grows out of A, it also maintains a degree of independence.[37]

Given how trikāla operates, in addition to this internal 2-expansion, it would be reasonable to expect several levels of uniform expansion or contraction of the entire melody. As such, one can easily calculate several possible integer expansions of the (4, 6, 0) interval spanning the melody using Proposition 2—recall that for any positive integer *k*, the *k*-expansion of (x, y, a) is given by (kx, y, ka):

$$(4, 6, 0)_{k=2} \rightarrow (2 \cdot 4, 6, 2 \cdot 0) = (8, 6, 0);$$
$$(4, 6, 0)_{k=3} \rightarrow (3 \cdot 4, 6, 3 \cdot 0) = (12, 6, 0);$$
$$(4, 6, 0)_{k=4} \rightarrow (4 \cdot 4, 6, 4 \cdot 0) = (16, 6, 0).$$

In fact, all three of these macro-expansions are manifested in the performance, as Figures 18, 19, and 20 illustrate. Figure 18 presents a 2-expansion that immediately follows the

37. In addition to rhythmic/metric considerations, note the similarity between the openings of A and B in terms of the pitches (*swaras*) employed.

first A–B melodic statement. The original melody appears in the upper bracketed region, while the 2-expanded melody appears below, spanning the interval

$$(8, 6, 0) = 4(1, 1, 0) + 2(2, 1, 0).$$

Significantly, all *Y*-downbeat shifts are neutralized to zero at this level of expansion, meaning that every *Y*-measure at this level must begin on beat 2 of the tāḷa.

Figure 19 presents the passage that follows the 2-expanded melody. Observe that this new passage is a 3-expanded variant of the original melody, spanning the interval

$$(12, 6, 0) = 4(1\ 1/2, 1, +3\ 1/2) + 2(3, 1, 0).$$

In this case, while the *Y*-downbeat within the B phrase remains stable, the *Y*-downbeat of A has begun shifting again, each time by half of a tāḷa cycle (+3 1/2). Moreover, as demonstrated by the *X*-coordinates, each *Y*-measure of A is now 1 1/2 āvartas long, so for the first time, the *Y*-measures of A surpass a full tāḷa cycle.

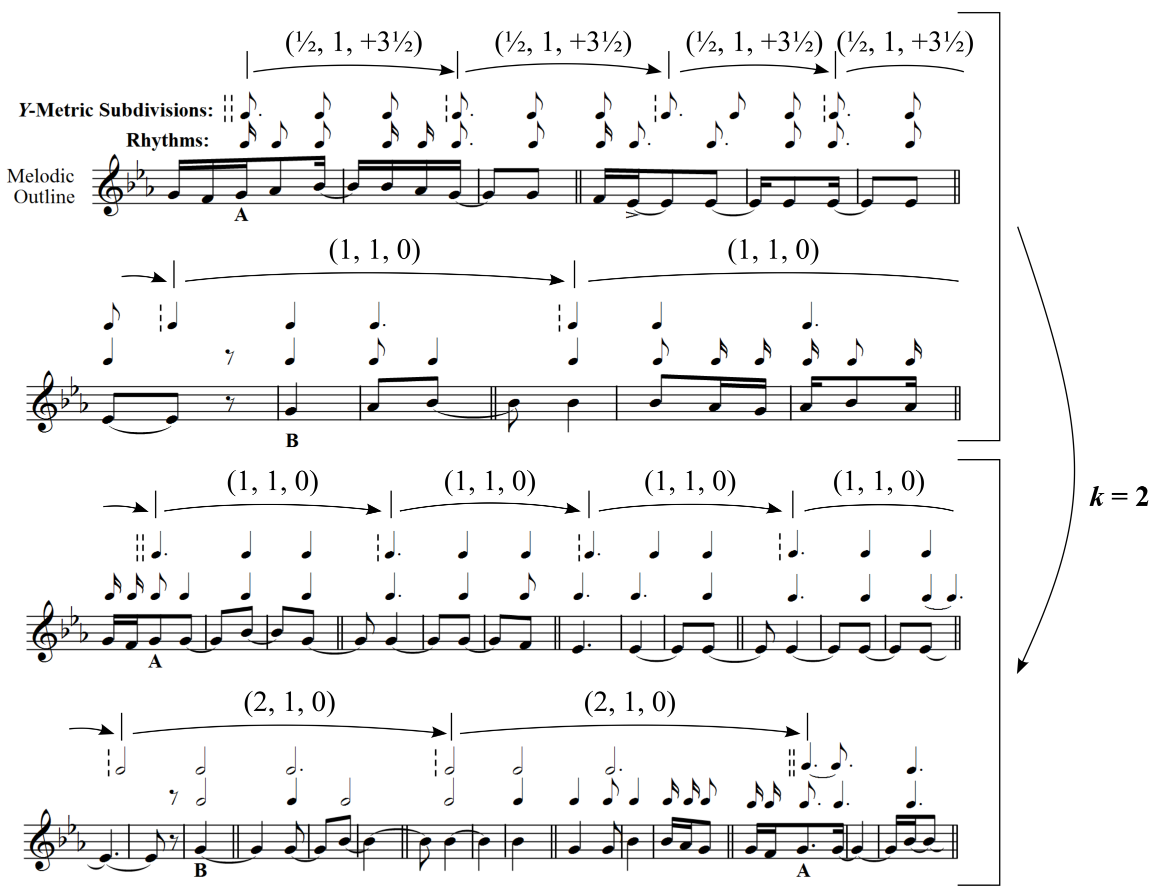


Figure 18. *Met*-intervallic 2-expansion of opening interval: (8, 6, 0) = 4(1, 1, 0) + 2(2, 1, 0).

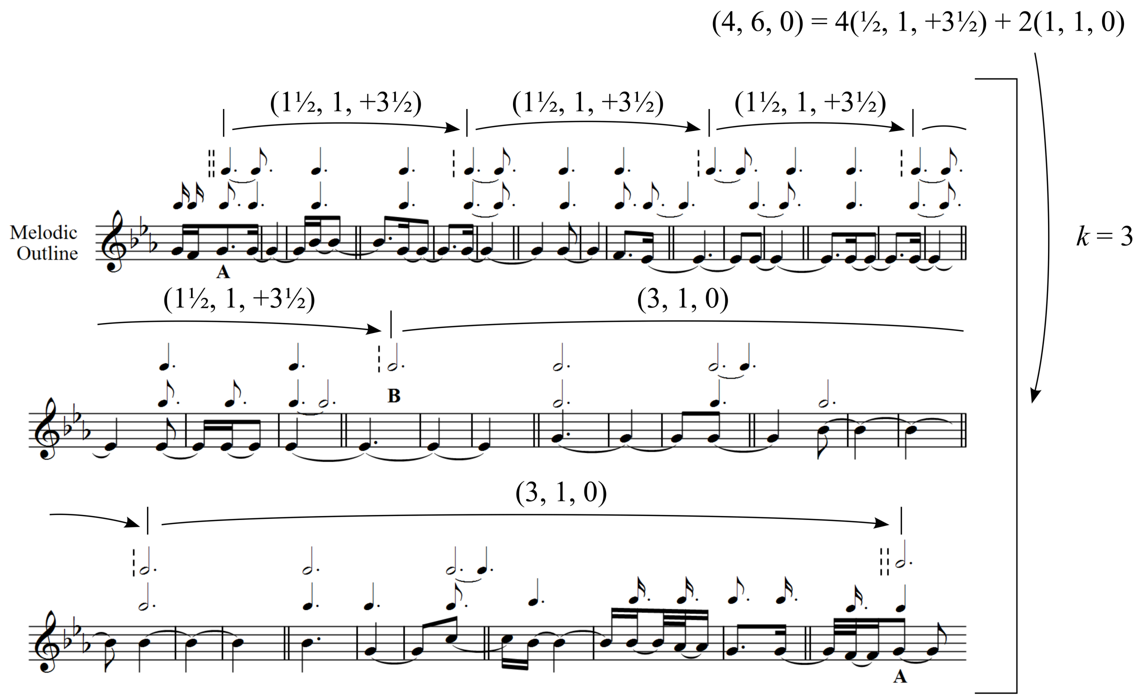


Figure 19. *Met*-intervallic 3-expansion of opening interval: (12, 6, 0) = 4(1 1/2, 1, +3 1/2) + 2(3, 1, 0).

Finally, Figure 20 demonstrates the 4-expansion of the pallavi melody following the passage in Figure 19. Here the expanded melody's overall *Met* interval is

$$(16, 6, 0) = 4(2, 1, 0) + 2(4, 1, 0).$$

This passage marks the apex of the successive intervallic expansions; B, in particular, now has extremely long *Y*-measures, each lasting four full tāḷa cycles. Additionally, all *Y*-downbeat shifts are again neutralized within both A and B phrases. Audio Example 5 presents the 3- and 4-expansions of the pallavi melody. While listening, please note the locations of the claps marking the second and third angas of each āvarta and observe the evolving relationship between the tāḷa-bound claps and the melody.

Figure 21 summarizes the progress of the melodic expansions thus far and indicates the contractions that will follow. The top half of the diagram presents the opening melodic interval, (4, 6, 0), and the 2-, 3-, and 4-expansions that follow. I have also included an alternative conception of these expansions: if expansion values are calculated according to consecutive intervals rather than always in reference to the opening interval, the initial 2-expansion is followed by a 1 1/2-expansion and a 1 1/3-expansion. Note that from this perspective, the expansion factors are steadily decreasing.[38] The bottom half of the diagram summarizes the intervallic transformations over the remainder of the trikāla excerpt. In this case, a series of

38. If *k*-expansions of (4, 6, 0) are allowed for arbitrarily large *k*, then the corresponding *k*-values of the *consecutive* interval expansions will, in fact, tend to $k = 1$.

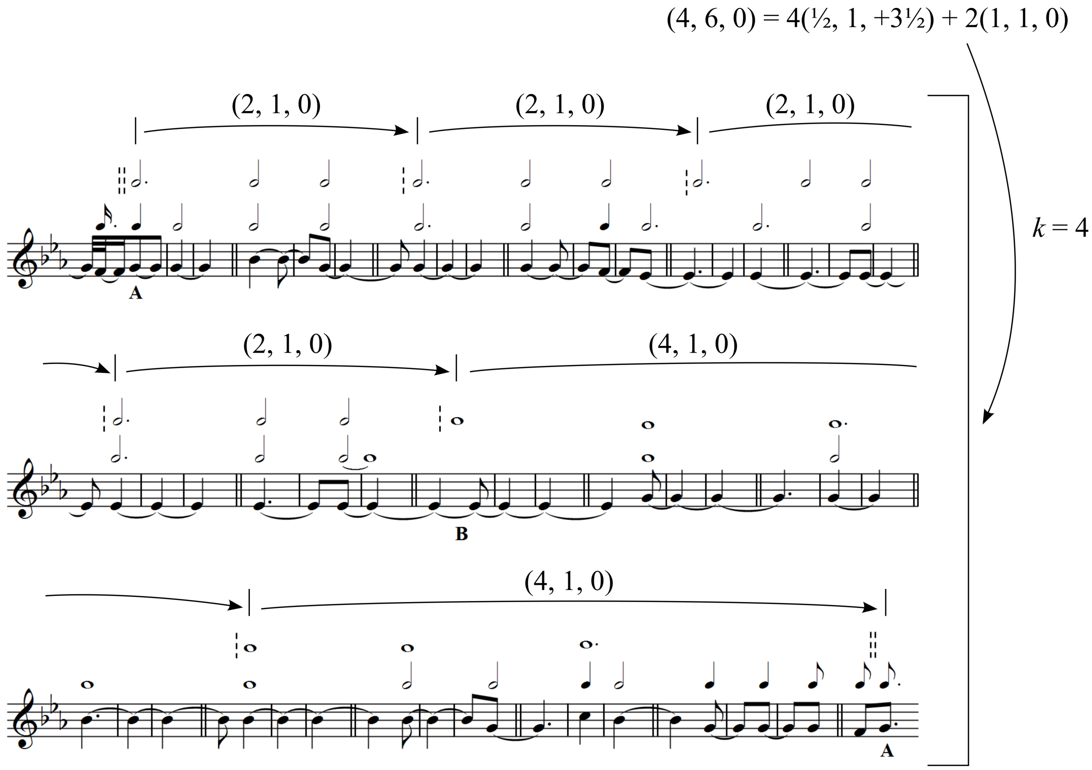


Figure 20. *Met*-intervallic 4-expansion of opening interval: (16, 6, 0) = 4(2, 1, 0) + 2(4, 1, 0).

Audio Example 5. 3- and 4-expansions of the basic A–B melody.

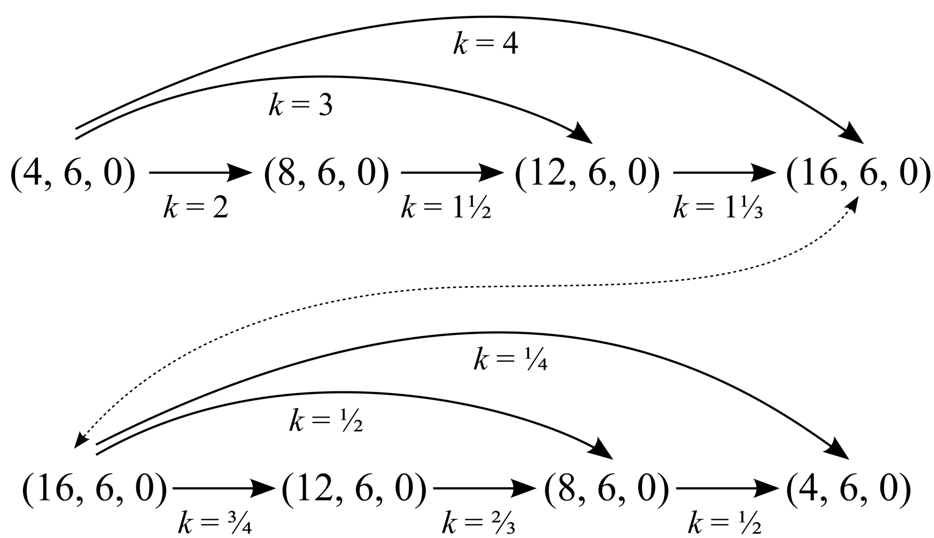


Figure 21. Successive expansions and contractions of the opening interval, (4, 6, 0) = 4(1/2, 1, +3 1/2) + 2(1, 1, 0), across the trikāla portion of the RTP performance.

successive contractions undoes all of the k-expansions and ultimately restores the original (4, 6, 0) interval.

Amidst all of these macro-expansions and contractions, however, the aforementioned internal 2-expansion from A to B is still active, albeit at changing levels of pulse. Additionally, it is important not only to consider the various k-values, but to observe what is happening to the *Met* intervals themselves. Namely, note that only the X-coordinate is changing across these expansions and contractions, as the A–B melody always consists of six Y-measures and the net Y-downbeat shift is always zero, even if internal melodic units enact nonzero shifts.

In fact, an even stronger statement can be made concerning the net Y-downbeat shifts: for *any* positive integer k, the k-expansion of the A–B melody must enact an overall Y-downbeat shift of zero. This follows from Proposition 2, as the "0" coordinate of (4, 6, 0) becomes $k \cdot 0 \equiv 0 \pmod 7$ for any value of k. Thus, whether k is equal to 2 or 2,000, the net Y-downbeat shift of the k-expanded melody will be zero, even within the asymmetrical Miṣra Cāpu tāḷa (7-meter). As a result, any positive integer expansion of the melody must, like the original melody, begin and end on beat 2 of the tāḷa.

While the zero-valued Y-downbeat shifts, predictable scaling of Y-decompositions, and number of Y-measures in the melody (six) represent retained properties in the performance's melodic expansions and contractions, other aspects of the relationship between melody and tāḷa change dramatically. Specifically, a special intervallic decomposition I henceforth call the *anga decomposition* is often drastically altered in expansions and contractions. The anga decomposition functions similarly to the Y-decomposition, breaking a *Met* interval into subintervals. However, rather than decomposing intervals at Y-downbeats, this decomposition breaks up intervals according to anga boundaries, making it uniquely suited to analysis of Indian classical music.[39]

Figure 22 presents the anga decompositions of the four (1/2, 1, +3 1/2) intervals in the unexpanded melody's A phrase. Observe that intervallic decompositions occur anytime a solid bar line (single or double) appears in the transcription. Thus, while each Y-measure generates the same *Met* interval (1/2, 1, +3 1/2), these intervals anga-decompose non-uniformly. Specifically, while the first and third intervals decompose as

$$(1/2, 1, +3\ 1/2) = (2/7, 4/7, 0) + (3/14, 3/7, +3\ 1/2),$$

the second and fourth decompose as

$$(1/2, 1, +3\ 1/2) = (1/14, 1/7, 0) + (2/7, 4/7, 0) + (1/7, 2/7, +3\ 1/2).$$

One immediately visible difference in the decompositions is the number of subintervals—two in the first case versus three in the second case. Moreover, while both

39. For more on the anga decomposition as a component of *Met*-based analyses of Carnatic music, see Wells (2015a, 235–38).

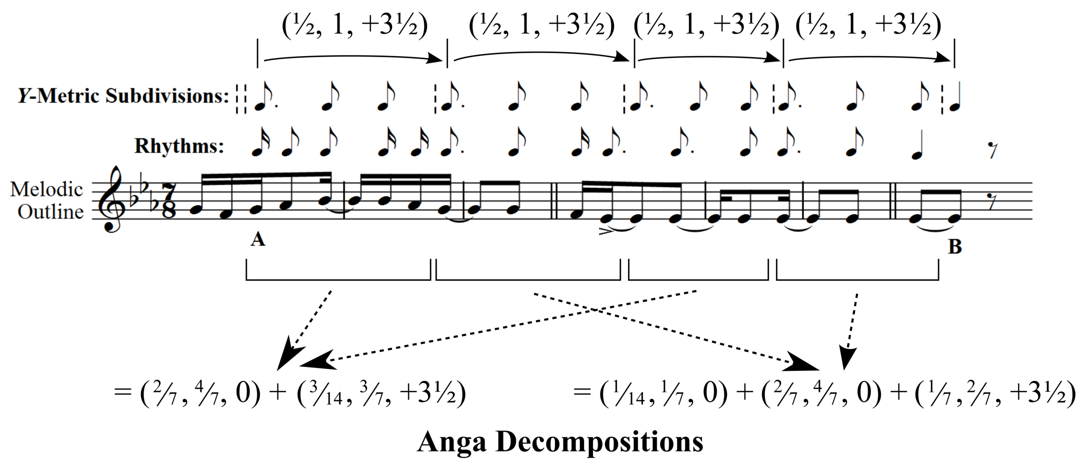


Figure 22. *Met*-intervallic span of A phrase, with basic intervals anga-decomposed.

decompositions contain the subinterval (2/7, 4/7, 0), the remaining intervallic span (3/14 *X*-measures) is undivided in the first case and split asymmetrically in the second case.

In contrast, consider the 2-expansion of A, shown in Figure 23. While the anga decompositions of the unexpanded *Y*-measures exhibit marked differences, each *Y*-measure of 2-expanded A has the same anga decomposition,

$$(1, 1, 0) = 3(2/7, 2/7, 0) + (1/7, 1/7, 0).$$

Thus, the 2-expansion not only neutralizes the *Y*-downbeat shifts (+3 1/2 → 0), as discussed previously, but regularizes the relationship between melody and tāḷa. The anga decompositions therefore highlight new rhythmic/metric implications that the *Y*-decompositions missed.

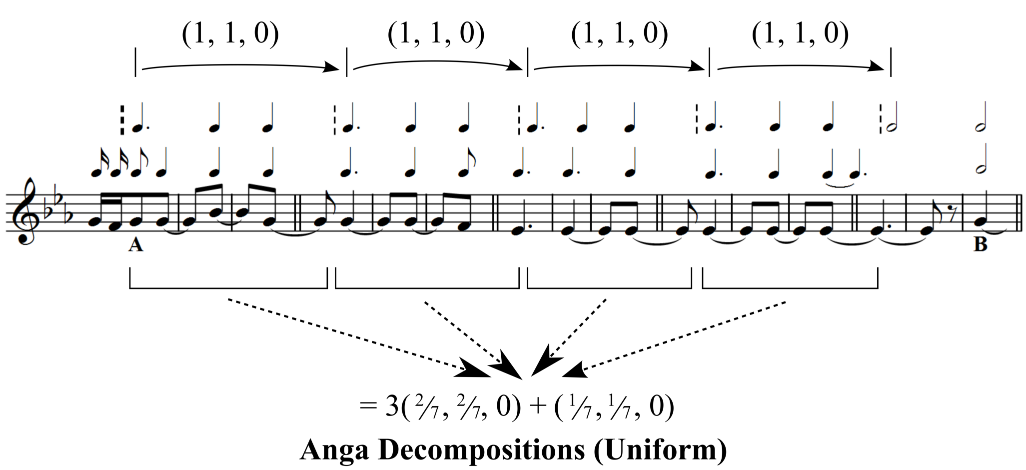


Figure 23. 2-expanded variant of A, with basic intervals anga-decomposed.

Next, consider the effect of the 2-expansion on the B phrase of the melody. Figure 24 presents unexpanded B with *Met*-intervallic annotations. As shown below the transcription, both *Y*-measures of B anga-decompose to yield

$$(1, 1, 0) = 3(2/7, 2/7, 0) + (1/7, 1/7, 0),$$

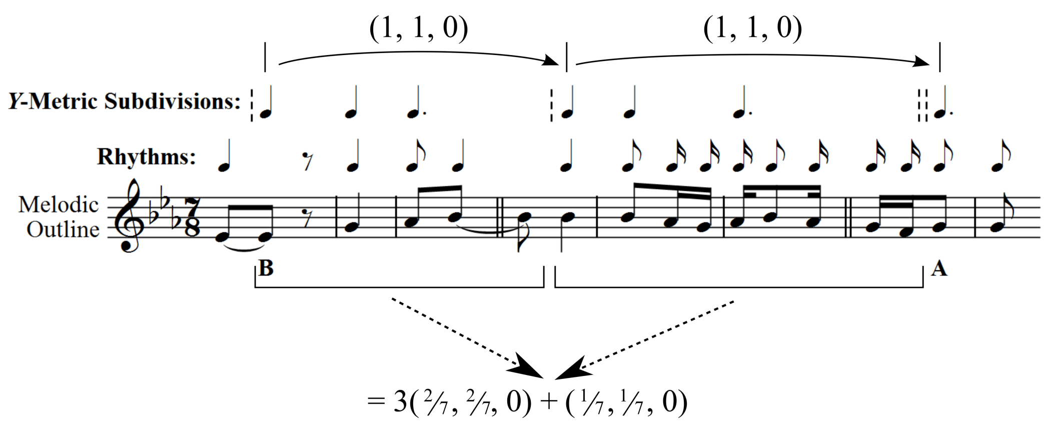
which is the same anga decomposition that characterized 2-expanded A. Before considering 2-expanded B, recall that the *Y*-measures of unexpanded A had varying anga decompositions, while all *Y*-measures of 2-expanded A had the same anga decomposition. Thus, the 2-expansion transformed a non-uniform anga decomposition into a uniform one, representing a local reduction in the complexity of the phrase-tāḷa relationship.

A similar decrease in intervallic complexity does not occur when B is 2-expanded, however, as Figure 25 illustrates. In this case, rather than a "non-uniform-to-uniform" process, the *Y*-measures of 2-expanded B still have identical anga decompositions, implying a "uniform-to-uniform" process. Thus, whether B is 2-expanded or unexpanded, the *Y*-measures have a constant relationship with the tāḷa. The specifics of the decomposition change drastically in the expansion, though, for there are now seven components comprising three distinct *Met* intervals (due to repetitions):

$$(2, 1, 0) = 3(2/7, 1/7, 0) + (3/7, 3/14, 0) + 2(2/7, 1/7, 0) + (1/7, 1/14, 0).$$

Therefore, the 2-expansion operation substantially increases the *Met*-intervallic complexity of B, despite the apparently subtle change in B's *Y*-decomposition from 2(1, 1, 0) to 2(2, 1, 0).

Unsurprisingly, as A and B are expanded two more times (3-expansion and 4-expansion), the numbers of components in the anga decompositions continue to increase, with the 4-expansion of B containing the most components (thirteen). Table 1 provides the *Met*-intervallic details. Notably, while the *Y*-measures of B continue to decompose uniformly at the new



Anga Decompositions (Uniform)

Figure 24. *Met*-intervallic span of unexpanded B, with basic intervals anga-decomposed.

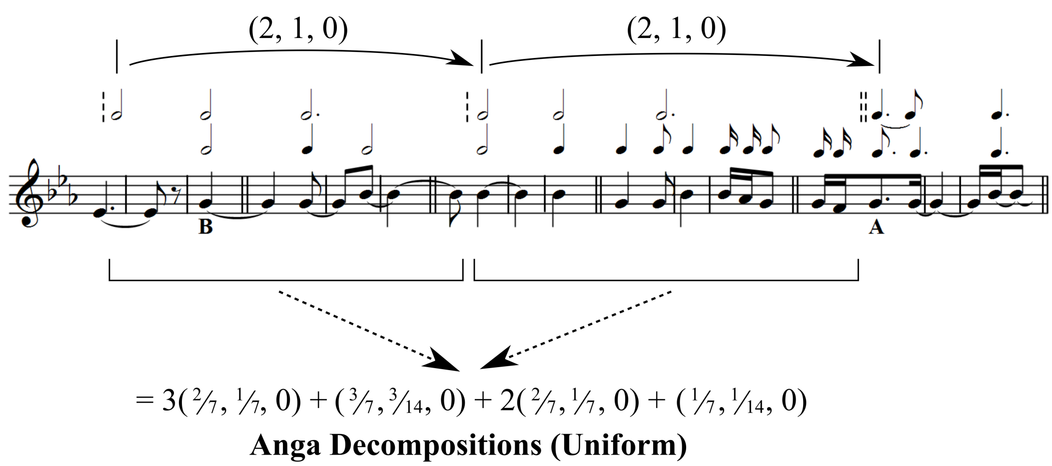


Figure 25. 2-expanded variant of B, with basic intervals anga-decomposed.

Intervals to be Decomposed	Anga Decompositions
3-expanded A: first and third *Y*-measures	**(1 1/2, 1, +3 1/2)** = 3(2/7, 4/21, 0) + (3/7, 6/21, 0) + (3/14, 3/21, +3 1/2)
3-expanded A: second and fourth *Y*-measures	**(1 1/2, 1, +3 1/2)** = (1/14, 1/21, 0) + (2/7, 4/21, 0) + (3/7, 6/21, 0) + 2(2/7, 4/21, 0) + (1/7, 2/21, +3 1/2)
3-expanded B: all *Y*-measures	**(3, 1, 0)** = 3(2/7, 2/21, 0) + (3/7, 3/21, 0) + 2(2/7, 2/21, 0) + (3/7, 3/21, 0) + 2(2/7, 2/21, 0) + (1/7, 1/21, 0)
4-expanded A: all *Y*-measures	**(2, 1, 0)** = 3(2/7, 1/7, 0) + (3/7, 3/14, 0) + 2(2/7, 1/7, 0) + (1/7, 1/14, 0)
4-expanded B: all *Y*-measures	**(4, 1, 0)** = 3(2/7, 1/14, 0) + 3[(3/7, 3/28, 0) + 2(2/7, 1/14, 0)] + (1/7, 1/28, 0)

Table 1. Anga decompositions of the *Met* intervals defining 3-expanded and 4-expanded variants of A and B.

expansion levels, the *Y*-measures of A do not. Specifically, while 4-expanded A consists of *Y*-measures that all anga-decompose in the same way, 3-expanded A returns to non-uniformity, thereby recalling unexpanded A. Thus, while the successive expansions of A and B express increasing tension with the tāḷa due to their growing numbers of anga decomposition components and transforming interval values, the A phrase evinces an additional sense of instability due to its alternation between uniform and non-uniform decompositions. This instability continues to play a role even as the expansions yield to contractions in the latter portion of the trikāla section (see Figure 21).

The final melodic material the listener hears is the metrically unstable A phrase in its original, unexpanded form with non-uniform internal anga decompositions. Thus, it would be reasonable to expect the performers to provide some sort of metric closure to the trikāla presentation in anticipation of the swara kalpana section that follows, in which the musical focus shifts back to melody and rāga. In fact, after the final A statement, the mridangam takes over the metric narrative, further amplifying the metric tension through a brief cadenza. Using *Met* can not only help describe this amplification in precise, quantifiable terms, but can yield some performative insights. Figure 26 presents a transcription of this cadenza with analytical annotations.[40] My notation of the mridangam part (after Widdess 1977) begins at the start of the cadenza, for while the mridangam player (T. Ranganathan) has been playing up to this point, his role has primarily been accompanimental. The excerpt can be heard in Audio Example 6.

The beginning of the transcription depicts the end of A and the final two generated (1/2, 1, +3 1/2) intervals. These intervals allow a sort of dovetailing between the melody and the mridangam cadenza, as the cadenza begins with five statements of (1/2, 1, +3 1/2). However, this key interval has been recontextualized: rather than generating alternating *Y*-downbeats on beats 2 and 5 1/2, as the original A phrase did, the *Y*-downbeats now alternate between 3 1/2 and 7.[41] In one sense, the passing of (1/2, 1, +3 1/2) to the mridangam suggests a conclusion to the metric narrative that has defined the trikāla portion of this performance, as this interval served as a point of departure for the progressive metric transformations. Its return therefore brings the listener full circle.

Mathematically, though, there is no way Ranganathan can end the cadenza on sam (beat 1 of the tāḷa) without breaking free of this characteristic interval. Thus, starting in tāḷa cycle 71, he begins incorporating *Y*-measures that have been shortened by half of a beat (one "sixteenth note"), generating repetitions of the new interval (3/7, 1, +3). This seemingly small rhythmic

40. While Widdess (1977, 74) hears a series of 7-units partitioned as 3+2+2 starting where my mridangam notation begins, I hear a 2+2+3 partition beginning where indicated. This interpretation permits hearing five "6/16" *Y*-measures from tāḷa cycle 71 through the end of the excerpt rather than a series of eleven "3/16" *Y*-measures, thus highlighting symmetric aspects of the cadenza (five "7/16" *Y*-measures + five "6/16" *Y*-measures). Additionally, the 2+2+3 partitioning recalls the consistent metric subdivisions of A throughout the trikāla improvisation.

41. Between the end of A (disregarding the sustained "sa" pitch) and the start of the mridangam cadenza, a *Y*-downbeat shift of +1 1/2 somehow occurs. There are essentially two explanations for this shift. One is that it is accomplished via the interval (3/14, 1, +1 1/2), which bridges the gap between the melodic and mridangam intervals in the top line of Figure 26. Effectively, this interval spans a mini *Y*-measure of 1½ beats. The second option is to hear this "mini-measure" not as an independent *Y*-measure, but as an extension of the previous (1/2, 1, +3 1/2) interval—i.e., the final *Y*-measure of A lasts longer than one initially expects. This would mean that the latter interval becomes a (5/7, 1, +5) leading directly into the start of the mridangam cadenza.

change is significant in the context of 7-meter, for it allows the *Y*-downbeat to begin cycling through all of the integer downbeat locations.[42] Moreover, this is the first recurring, *Y*-

Figure 26. Mridangam cadenza at end of trikāla section. Transcription after Widdess (1977), metrically reinterpreted and with rhythmic and *Met*-intervallic annotations by the author.

[Audio Example 6.](#) Mridangam cadenza at end of trikāla section.

42. Because the new interval has a *Y*-downbeat shift value of +3, iterating this interval will yield shifts of the form $+3x$ (mod 7), where x is a positive integer. An elementary result in mathematical group theory is that any group with prime order (such as the additive integers mod 7) is cyclic and can be generated by any single group element (see, for instance, Fraleigh and Katz 2003, 100–101, Corollary 10.11). Thus, successive +3 shifts—namely, taking $x = 1$, $x = 2$, etc., in the above formula—will cycle through all possible integer-valued *Y*-downbeat shifts in 7-meter, resulting in a corresponding cycling through all possible *Y*-downbeat locations.

measure-spanning interval in the trikāla section that cannot be generated by an integer expansion of (1/2, 1, +3 1/2). As such, it exhibits a certain independence from the performance's other characteristic intervals.

Additionally, observe that the series of new intervals begins on beat 7 of the tāḷa. In order for *X*- and *Y*-downbeats to realign at the end of the trikāla section, a net *Y*-downbeat shift of +1 is necessary. The central question, then, is how many iterations of (3/7, 1, +3) are necessary to accomplish this shift. Basic modular arithmetic could certainly help answer this question; namely, achieving a value of +1 modulo 7 only requires adding +3 to itself five times. In fact, Ranganathan iterates the new phrase length exactly five times in the performance, as shown in Figure 26. Ranganathan's experience undoubtedly allows him to make this determination in the midst of performance using traditional methods. In particular, Iyer (1988) describes how a drummer might quickly determine, from any location in the tāḷa, how many phrases of a given length and speed are necessary to conclude on sam. Iyer lists numerous numerical formulas that a drummer can internalize to arrive on sam artfully via expressive rhythmic shapes.

Whether one uses modular arithmetic or traditional formulas, however, it is clear that five iterations of the six-"sixteenth-note" phrase length are needed to arrive back on sam. Intervallically, this yields 5(3/7, 1, +3) = (2 1/7, 5, +1) across all five phrases. The resulting cumulative interval shows that the arrival on sam occurs just over two tāḷa cycles after the six-pulse phrases began, accompanied by a net +1 *Y*-downbeat shift (as desired). In the spirit of Lewin, one can view the component (3/7, 1, +3) intervals not as mere statistics, but as signifiers in an overall metric narrative. While (1/2, 1, +3 1/2) has opening and closing function in the trikāla section of the RTP performance, (3/7, 1, +3) suggests new functional meaning. Notably, this interval has not occurred previously in the trikāla region, so its occurrence just before the swara kalpana suggests dual closing and transition functions.

The overarching metric story during the trikāla section, then, involves successive expansions of (1/2, 1, +3 1/2), followed by successive contractions to restore the original interval, and finally concluding with a new (3/7, 1, +3) interval that drives this portion of the performance to a close. While certain aspects of the original (1/2, 1, +3 1/2) interval are preserved (e.g., the 1-*Y*-measure span and the seven-pulse length at some metric level) in the process of expansion and contraction, other key aspects are transformed. For instance, the *Y*-downbeat shift value is neutralized to zero in all performed expansions except for $k = 3$, while the internal anga decompositions dramatically increase in complexity as k increases. With respect to the transforming anga decompositions for both the A and B phrases, the 4-expanded version of B, shown in Figure 20, marks an apex. Namely, the phrase's unassuming (4, 1, 0) interval—the 8-expansion of (1/2, 1, +3 1/2)—yields a staggering thirteen-component anga decomposition (see Table 1) whose component intervals are, moreover, distributed asymmetrically. The *Met* intervals reveal, then, that the climactic 4-expanded B phrase, whose melodic content is deceptively simple, is one of the most metrically intense portions of the performance. More generally, the intervallic narrative precisely depicts the evolving relationships between tāḷa and melodic phrasing in this RTP performance, representing both the local tensions between melody and tāḷa and the broader metric progressions that define the trikāla section.

Conclusion

A few final thoughts are in order. First, with respect to the latter analysis, while local 2-expansions from A to B occur throughout the trikāla performance, originally via the transformation

$$(1/2, 1, +3\ 1/2)_{k=2} = (1, 1, 0),$$

these mini-expansions later acquire new meaning (i.e., *Met* intervals with different sorts of properties) through macro-expansions and contractions. In particular, when the melody is uniformly expanded by 2, 3, and 4, the following new local transformations from A to B result:

$$(1, 1, 0)_{k=2} \longrightarrow (2, 1, 0);$$
$$(1\ 1/2, 1, +3\ 1/2)_{k=2} \longrightarrow (3, 1, 0);$$
$$(2, 1, 0)_{k=2} \longrightarrow (4, 1, 0).$$

As such, the contextual effect of the 2-expansion operation evolves as local pulse units change via the macro-expansions.

The RTP analysis also demonstrated a striking balance between stability and change as the trikāla improvisation progresses. In particular, while the A–B melody's cumulative *Y*-downbeat shift of zero and predictable *Y*-decompositions at various pulse levels represent stability, the chaotic, unpredictable effect of the expansions/contractions on anga decomposition suggests a dynamic instability. Moreover, while the expansions and contractions have predictable intervallic consequences, the resulting intervals are new (in the context of this performance) and specify transformed relationships between tāḷa and melody, even while being derived from old intervals.

The alankāram exercises suggested a similar balance between stability and change. As the performer moves from first speed through second and third speeds, the component phrases in each exercise retain their melodic shape and swara structure, while the evolving *Met* intervals illuminate new contextual functions for each phrase with respect to the tāḷa. These phrase functions are not mere theoretical abstractions, but reflect key aspects of the performance experience.

In this regard, I present a few closing remarks about *Met* theory and its appropriateness for analyzing Carnatic music. First, while it might be tempting to conceive of the metric expansions and contractions as independent of the tāḷa, Indian tradition holds that all rhythmic formations, regardless of complexity, must be understood in relation to the tāḷa.[43]

43. Ramanathan and Venkataram (1997), in an analysis of the varnam "Ninnukori," highlight how the "simultaneous presence of the basic rhythmic pattern of 4 established by the tala accent and the 5, 6 etc. formed

Thus, the *Met* intervals, which address not only the tāḷa and melodic/rhythmic layers (*X*- and *Y*-coordinates, respectively), but also the layers' interrelations (*Y*-downbeat shift coordinate), depict the multiple simultaneous dimensions performers must have under constant control. Indeed, while one could, perhaps, criticize the three-coordinate *Met* intervals as unnecessarily complex, I would argue that if even one intervallic coordinate were missing, part of the musical picture would be unrepresented. In particular, without the *X*-coordinate, the intervals ignore the flow of the unfolding tāḷa; without the *Y*-coordinate, the intervals miss the phrases and groupings that create expressive tension with the tāḷa; and without the *Y*-downbeat shift coordinate, there is no explicit representation of the interactions between the melodic/rhythmic and tāḷa layers. Each of these aspects is essential for representing the multidimensional rhythmic/metric experience in Carnatic music.

While the theory and analyses in this article may be useful in their own right, they also suggest several directions for further study. First, one could use *Met* to investigate Carnatic trikāla usages in further contexts—not only in RTP form and the alankāram exercises, but also musical forms such as *geetham*s and *varṇam*s and improvised percussion solos.[44] One might also study the broader theoretical possibilities for implementing trikāla technique in the many available tāḷas: what are the intervallic consequences of expansion/contraction by various factors, and how are anga decompositions affected? Moreover, how are the same intervals affected when *k*-expanded in different tāḷas? Also revealing would be a corpus study of common *Met* intervals appearing in Carnatic music and their most frequently occurring expanded and contracted forms. In sum, the current study has provided but a glimpse into the diverse possibilities for analyzing pulse transformation in Carnatic music using *Met*. It is my hope that further study will not only address the additional avenues of exploration suggested here, but will generate new questions about the interactions between rhythm, melody, and tāḷa in this remarkably rich music.

ACKNOWLEDGMENTS

I thank everyone who has provided feedback or influenced this project in some way, whether directly or indirectly. In particular, I thank Robert Morris for his indispensable feedback in the early stages of this project; Sangeetha Agarwal for her valuable insights into Carnatic vocal performance and pedagogy; Rohan Krishnamurthy for equally valuable insights into Carnatic percussion performance and pedagogy; and the anonymous reviewers for their helpful critiques of this work.

by the melodic accents bestow a peculiar charm on the over-all rhythmic colour of the song" (65). In other words, it is not the melodic accents on their own that are of greatest musical interest, but their counterpoint with the tāḷa accents.

44. The *geetham* and *varṇam* are forms intended to develop one's musical technique, much like the Western classical étude. In addition to helping a student practice characteristic rāga phrases in context, these compositions aid in the student's understanding of rhythm and time, as they are practiced (and, in the case of varṇams, performed) in multiple speeds over constant tāḷa. See Subramaniam and Subramaniam (1995, 80–82) and Pesch (1999, 84–85).

REFERENCES

Cady, Henry L. 1983. Review of *A Generative Theory of Tonal Music*, by Fred Lerdahl and Ray Jackendoff. *Psychomusicology: A Journal of Research in Music Cognition* 3(1): 60–67. https://doi.org/10.1037/h0094250.

Field, Garrett. 2018. "Improvising Rhythmic-Melodic Designs in South Indian Karnatak Music." *Analytical Approaches to World Music* 6(2). http://aawmjournal.com/articles/2018a/Field_AAWM_Vol_6_2.html.

Fraleigh, John B., and Victor Katz. 2003. *A First Course in Abstract Algebra*. 7th ed. Boston: Addison-Wesley.

Iyer, S. Rajagopala. 1988. *Sangeetha Akshara Hridaya: A New Approach to Tala Calculations*. Bangalore: Gaanarasika Mandali.

Krishna, T. M. 2013. *A Southern Music: The Karnatic Story*. Noida: HarperCollins India.

Lerdahl, Fred, and Ray Jackendoff. 1983. *A Generative Theory of Tonal Music*. Cambridge, MA: MIT Press.

Lewin, David. 1987. *Generalized Musical Intervals and Transformations*. New Haven: Yale University Press.

London, Justin. 1997. "Lerdahl and Jackendoff's 'Strong Reduction Hypothesis' and the Limits of Analytical Description." *In Theory Only* 13 (1–4): 3–29.

———. 2004. *Hearing in Time: Psychological Aspects of Musical Meter*. Oxford: Oxford University Press.

Mirka, Danuta. 2009. *Metric Manipulations in Haydn and Mozart: Chamber Music for Strings, 1787–1791*. Oxford: Oxford University Press.

Morris, Robert. 1987. *Composition with Pitch-Classes: A Theory of Compositional Design*. New Haven: Yale University Press.

———. 1998. "Sets, Scales, and Rhythmic Cycles: A Classification of Talas in Indian Music." Unpublished manuscript. Eastman School of Music, University of Rochester. http://lulu.esm.rochester.edu/rdm/pdflib/talapaper.pdf.

———. 2000. "Crowns: Rhythmic Cadences in South Indian Music." Paper presented at Annual Meeting of the Society for Music Theory, Toronto. November 2.

———. 2006. "Architectonic Composition in South Indian Classical Music: The 'Navaragamalika Varnam.'" In *Analytical Studies in World Music*, edited by Michael Tenzer, 303–31. Oxford: Oxford University Press.

Nelson, David P. 1991. "*Mṛdaṅgam* Mind: The *Tani Āvartanam* in Karṇāṭak Music." PhD diss., Wesleyan University.

———. 2000. "Karnatak Tala." In *The Garland Encyclopedia of World Music, Vol. 5: South Asia: The Indian Subcontinent*, edited by Alison Arnold, 138–61. London: Garland Publishing.

———. 2008. *Solkattu Manual: An Introduction to the Rhythmic Language of South Indian Music*. Middletown, CT: Wesleyan University Press.

———. 2019. *Konnakkol Manual: An Advanced Course in Solkattu*. Middletown, CT: Wesleyan University Press.

Nestke, Andreas, and Thomas Noll. 2001. "Inner Metric Analysis." *Tatra Mountains Mathematical Publications* 23(3): 91–111.

Ng, Samuel. 2005. "A Grundgestalt Interpretation of Metric Dissonance in the Music of Johannes Brahms." Ph.D. diss., University of Rochester.

———. 2006. "The Hemiolic Cycle and Metric Dissonance in the First Movement of Brahms's Cello Sonata in F Major, Op. 99." *Theory and Practice* 31: 65–95.

Pesch, Ludwig. 1999. *The Illustrated Companion to South Indian Classical Music*. Delhi: Oxford University Press.

Ramanathan, N., and Bangalore K. Venkataram. 1997. *Essays on Tala and Laya*. Bangalore, India: Percussive Arts Centre.

Ravikiran, Chitravina N. 2012. *Perfecting Carnatic Music, Level I*. Chennai: International Foundation for Carnatic Music.

Śarma, Ākeḷla Mallikārjuna. 1992. *Permutative Genius in Tala (Prastara) in Indian Music.* Hyderabad: Telugu University.

Schachter, Michael L. 2015. "Structural Levels in South Indian Music." *Music Theory Online* 21(4). http://www.mtosmt.org/issues/mto.15.21.4/mto.15.21.4.schachter.html.

Subramaniam, L., and Viji Subramaniam. 1995. *Euphony: Indian Classical Music*. Chennai: Eastwest Books (Madras).

Subramanian, Srikumar K., Lonce Wyse, and Kevin McGee. 2011. "Modeling Speed Doubling in Carnatic Music." Paper presented at the International Computer Music Conference, University of Huddersfield, UK. July 31–August 5.

Temperley, David. 2001. *The Cognition of Basic Musical Structures*. Cambridge, MA: MIT Press.

Tenzer, Michael. 2011. "Temporal Transformations in Cross-Cultural Perspective: Augmentation in Baroque, Carnatic, and Balinese Music." *Analytical Approaches to World Music* 1(1): 152–75. http://aawmjournal.com/articles/2011a/Tenzer_AAWM_Vol_1_1.htm.

Viswanathan, T., and Matthew Harp Allen. 2004. *Music in South India: The Karnatak Concert Tradition and Beyond*. New York: Oxford University Press.

Viswanathan, T., and Jody Cormack. 1998. "Melodic Improvisation in Karnatak Music: The Manifestations of Rāga." In *In the Course of Performance*, edited by Bruno Nettl and Melinda Russell, 219–33. Chicago: University of Chicago Press.

Volk, Anja. 2008. "The Study of Syncopation Using Inner Metric Analysis: Linking Theoretical and Experimental Analysis of Metre in Music." *Journal of New Music Research* 37(4): 259–73. https://doi.org/10.1080/09298210802680758.

Wells, Robert L. 2015a. "A Generalized Intervallic Approach to Metric Conflict." PhD diss., University of Rochester.

———. 2015b. "*Tāḷa* and Transformation: A GIS Approach to Metric Conflict in South Indian Carnatic Music." Paper presented at Annual Meeting of the Society for Music Theory, St. Louis, MO. October 30.

———. 2017. "A Generalized Intervallic Approach to Metric Conflict in Liszt." *Music Theory Online* 23(4). http://mtosmt.org/issues/mto.17.23.4/mto.17.23.4.wells.html.

Widdess, D. R. 1977. "Trikāla: A Demonstration of Augmentation and Diminution from South India." *Musica Asiatica* 1: 61–74.

SOUND RECORDINGS

Kalyanaraman, Shivkumar. "Jagadananda Karaka." http://www.shivkumar.org/music/jagadananda.mp3. Audio recording.

Schiff, András. 1990. *Bach: English Suites, BWV 806–11*. Decca 421640. Two compact discs.

Subramaniam, L., and Palghat Mani Iyer. 2013. *On Record*. Subramaniam Entertainment/Viji Records 887516271199. Compact disc.

Viswanathan, T., L. Shankar, and T. Ranganathan. 1973. *Pallavi: South Indian Flute Music*. Nonesuch H–72052. LP.