The Hurrian Pieces, ca. 1350 BCE: Part Two—From Numbered Strings to Tuned Strings

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Interpretation of the Hurrian scores becomes 'musical' in a more usual sense if the numbered strings considered in the first part of this study are interpreted in terms of Mesopotamian tuning. (In what follows, passages that depend on the first installment of this study are provided with links.)

Interpreting numbered strings in terms of Mesopotamian tuning replaces relationships among numbered strings with relationships among tuned strings. Although one's construal of the Hurrian scores differs if numbered strings are interpreted in terms of Mesopotamian tuning, the string-number relationships discussed in the first part of this study are to a large extent isomorphic with the relationships that result from considering the way in which strings were tuned. That is, if numbered strings are interpreted in terms of Mesopotamian tuning, structural aspects of h.6 and statistical aspects of all 35 scores that the first part of this study framed in terms of numbered strings and string-pairs correspond to structural and statistical aspects framed in terms of tuned strings and string-pairs. In particular, interpreting numbered strings in terms of Mesopotamian tuning results in relationships between the 'generic' aspects of intervals, i.e., the numbers of steps that intervals span (Clough and Myerson 1985, 249), which correspond directly to relationships between the numbers identified with pairs of strings.

Interpreting numbered strings in terms of Mesopotamian tuning results additionally in relationships between the 'specific' aspects of intervals, i.e., the magnitudes of intervals that span particular numbers of degrees (as in the distinction between major and minor seconds), an aspect of structure that has no direct counterpart among relationships between numbered strings and string-pairs *per se*. Briefly, whereas 0-mod-7 string-pairs correspond to tuned unison/octave intervals, corresponding to 1-mod-7 string-pairs are two kinds of tuned 1-mod-7 interval, namely, larger and smaller; similarly, there are two kinds of tuned intervals that correspond to 2-mod-7 string-pairs, again, larger and smaller, and, finally, two kinds of tuned intervals that correspond to 3-mod-7 string-pairs, yet again, larger and smaller.

Without interpreting numbered strings in terms of Mesopotamian tuning there would be, in principle, an infinite number of ways in which strings and string-pairs in the Hurrian scores could be realized, each of which would correspond to a particular configuration of specific-interval magnitudes. Taking Mesopotamian tuning into account reduces these to twelve main possibilities. These twelve possibilities involve three ways in which string-pairs can be construed as generic intervals, and each of these generic construals comprises two ways in which particular string-pairs can be understood as specific intervals, namely, large and small or small and large, as well as two ways in which the strings that comprise these specific intervals can be arranged registrally, i.e., from higher to lower or from lower to higher. In contrast, previous studies of Mesopotamian music have considered only two of the twelve main possibilities: indeed, only very special cases of these two possibilities.

MESOPOTAMIAN TUNING

The basis of our knowledge of the string-pairs' tonal framework is the right column of the front, obverse side of cuneiform tablet U.7/80 (ca. 1850 BCE: Gurney 1968, 229-32; Kilmer 1971, 140; Gurney 1974, 74). This column preserves in fragmentary form the only known Mesopotamian formulation for tuning—or more precisely, re-tuning—a harp or lyre. U.7/80 employs terms that are specified in two other cuneiform tablets: the names of individual strings of a nine-string harp or lyre that are listed in U.3011 and the names of string-pairs that CBS 10996 identifies with pairs of U.3011's string-names and with pairs of numerals from 1 to 7. (Relevant details of U.3011 and CBS 10996 are discussed in the first part of this study.)

As transcribed and transliterated by Gurney (1968, 229-31), a consequence of the re-tuning formulation's paradigmatic structure is that the cumulative effect of the Akkadian term *zaku* (*izaku* in line 11) and its complement, *la zaku* , i.e., 'not *zaku*' (*la za[kat]* in line 9 and *la zaku[tam]* in line 18) ensures that in each of seven tunings one of the 3-mod-7 string-pairs is *la zaku* and the other six are *zaku*. The question of what *zaku* and *la zaku* refer to remains open in this study. Unless otherwise indicated for the sake of illustration, I construe *zaku* and *la zaku* merely as complementary terms concerning the specific sizes of the intervals that result when 3-mod-7 string-pairs are plucked (i.e., 14, 25, … 73): e.g., diatonic instances of such complementary terms would be 'perfect fourth' and 'augmented fourth.'

Figure 1 provides a spare translation of the nineteen lines of the re-tuning formulation that have survived in U.7/80. As two portions of the nineteen lines in U.7/80 have been controversial, question marks in Figure 1 indicate the uncertainty of their translation.

Figure 1. Translation of the obverse side of cuneiform tablet U.7/80 (=UET VII 74, ca. 1850 BCE).

1. 73
2. 3
3. 73
4. harp/lyre 73
5. 36
6. 6
7. 36
8. harp/lyre [3]6
9. 62 *la zaku*
10. 2 and 9
11. 62 *zaku*
12. ?
13. harp/lyre 62
14. touch 25
15. alter? 2 and 9
16. harp/lyre 36
17. harp/lyre 36
18. t[ouch] *la zaku* 62
19. alter? 6

Nonetheless, line 12 in Figure 1 is certainly a rubric that separates the preceding lines, of which only lines 1 to 11 in Figure 1 survive, from the following lines, of which only lines 13 to 19 in Figure 1 survive.

Theo J. Krispijn (1990, 15; cf. also Gurney 1994, 101-02) has considered the cuneiform characters in line 12 to be *nusuẖ[um]* ('tightening') and regards them as a colophon indicating that lines 1 to 11 involved tightening strings on an instrument in order to change the instrument's tuning by raising the pitches produced by particular strings. According to Krispijn, the word *tunasaẖma* ('tightened') should be added to lines 2, 6, and 10, and would have had the effect of raising the pitch of, respectively, strings 3, 6, and 2 and 9. Similarly, Krispijn has read the word I have rendered as 'alter?' in lines 15 and 19 as, respectively, *te[niema]* and *tenie[ma]* ('loosen').

In contrast, Crocker (1997, 190) has claimed that in the original cuneiform writing of line 12 the character at the end of the word Krispijn reads as *nusuẖ[um]* is broken and incomplete and the '*e*' Krispijn adds to what had been earlier transliterated as *teni[ma]* in line 19 is similarly illegible, occurring as it does in the break at the very edge of what remains of the original tablet. A consequence of Crocker's claim is that any translation more determinate than 'loosen' or 'tighten' in lines 15 and 19 would be speculative.

The Paradigmatic Structure Of U.7/80

Notwithstanding uncertainty concerning the problematic words indicated by question marks in Figure 1, one can discern an unambiguous pattern among the surviving lines of U.7/80: specifically, a paradigm whose parallel segments comprise four lines. For example, the 73 string-pair appears in lines 1 and 3; in parallel fashion, the 36 string-pair appears in lines 5 and 7, and the 62 string-pair in lines 9 and 11. Moreover, the 62 string-pair is followed by the Akkadian phrase *la zaku* in line 9 and by its complement, namely, *zaku*, in line 11, suggesting that *la zaku* should be supplied in lines 5 and 1 and *zaku* in lines 7 and 3. Consistent with this pattern, 73 appears in lines 3 and 4, so that one can supply the missing numeral '3' in line 8 to complete the parallelism with line 7. Furthermore, lines 4 and 8 begin with the word for a harp or lyre (*sammû*), suggesting that the line which originally preceded line 1 also referred to such an instrument.

Between consecutive four-line groups, there is a consistent numerical pattern modulo-7. In a mod-7 framework, if one adds 3 to the 73 string-pair of lines 1 and 3, the result is the 36 string-pair of lines 5 and 7, for 3+3 = 6 mod-7 and 7+3 = 3 mod-7. Similarly, adding 3 to the 36 of lines 5 and 7 results in 62 in lines 9 and 11: 6+3 = 9 = 2 mod-7 and 3+3 = 6 mod-7. Because of the consistency of this numerical pattern, one can amplify what remains of the portion that precedes line 12 by successively subtracting 3-mod-7 one four-line segment at a time, as in Figure 2. Figure 2 also shows the results of supplying '*zaku*,' '*la zaku*,' and 'harp/lyre' in such four-line segments.

Whereas 62 is followed by *la zaku* in line 26, 62 is followed by *zaku* in line 28; similarly, for 36 in lines 22 and 24, 73 in lines 18 and 20, and so forth. Moreover, whatever word might have been destroyed in line 27, i.e., the counterpart of the problematic words *te[niema]* and *teni?[ma]* in lines 15 and 19, line 27 as a whole can be understood as referring to a change in strings 2 and 9 (= 2 mod-7) that turns *la-zaku* 62 into *zaku* 62. The same is true for 6 with regard to 36, 3 with regard to 73, and so forth.

Figure 2. Lines 1 to 11 of the obverse side of U.7/80 amplified to comprise 28 lines, numbered 1 to 28, on the basis of the paradigmatic structure of lines 1 to 11 in Figure 1, above.

1. [harp/lyre 62]
2. [25 *la zaku*]
3. [5]
4. [25 *zaku*]
5. [harp/lyre 25]
6. [51 *la zaku*]
7. [1 altered?]
8. [51 *zaku*]
9. [harp/lyre 51]
10. [14 *la zaku*]
11. [4 altered?]
12. [14 *zaku*]
13. [harp/lyre 14]
14. [47 *la zaku*]
15. [7 altered?]
16. [47 *zaku*]
17. [harp/lyre 47]
18. 73 [*la zaku*]
19. 3 [altered?]
20. 73 [*zaku*]
21. harp/lyre 73
22. 36 [*la zaku*]
23. 6 [altered?]
24. 36 [*zaku*]
25. harp/lyre [3]6
26. 62 *la zaku*
27. 2 and 9
28. 62 *zaku*

As noted above, the fourth line of each four-line segment comprises the same pair of numerals as the first line of the next four-line segment. Moreover, the Akkadian terms that appear in the third lines of the four-line segments correspond to the names given to individual strings in U.3011. Consequently, changing, for example, the *la-zaku* 36 string-pair of line 22 into the *zaku* 36 string-pair of line 24 by altering the sixth string named in U.3011 can be understood as resulting in the '36 harp/lyre' of line 25. Similarly, changing the *la-zaku* 73 string-pair of line 18 into the *zaku* 73 string-pair of line 20 by altering the 3rd string named in U.3011 can be understood as resulting in the '73 harp/lyre' of line 21, and so forth.

As Hans Martin Kümmel (1970, 255-56) discerned, the pair of numbers in the first line of each four-line segment can be understood as referring not merely to a nine-string harp or lyre that had been re-tuned by altering a string (or, as in, line 27, two strings); as well, the pair of numbers in the first line of each four-line segment can be understood as the name of the resulting tuning of the harp or lyre. For instance, altering string 6 in line 23 not only changes the *la-zaku* 36 string-pair of line 22 into the *zaku* 36 string-pair of line 24, such an alteration also changes the '73 tuning' of a harp or lyre of line 21 into the '36 tuning' of the same harp or lyre in line 25, which is effectively no longer 'the same' because of the change to its tuning. In this way, the Akkadian terms for such string-pairs as 36 and 73 also serve as the names for tunings.

As a result of line 20, the 73 string-pair is *zaku* in the 36 tuning (line 25), for, unlike 36, it has not been changed in the meantime (lines 21 to 24). However, the opposite does not hold: the 36 string-pair is not *zaku* in the 73 tuning (line 21), for it is specified as *la zaku* (line 22). Conversely, in successive four-line segments the *zaku* string-pairs accumulate. For example, whereas the 62 string-pair is *la zaku* in the 36 tuning (line 26), the following string-pairs in the 36 tuning are still *zaku*, for they have not been altered since they were identified as *zaku* in the tunings that precede 36 in the paradigm:

36 (line 24)
73 (line 20)
47 (line 16)
14 (line 12)
51 (line 8)
25 (line 4)

Indeed, if the paradigm were continued by subtracting 3 mod-7, the four-line segment preceding line 1 (which can be enumerated as lines -3, -2, -1, and 0) would be the same as lines 25 to 28:

-3. harp/lyre [3]6
-2. 62 *la zaku*
-1. 2 and 9 altered?
0. 62 *zaku*

In other words, one could cycle through lines 1 to 28 by subtracting 3 (or adding 4). As the cycle of lines 1 to 28 is, in principle, infinite, each of the seven tunings will have accumulated six *zaku* string-pairs and one *la-zaku* string-pair.

Conversely, lines 13 to 19 of the original cuneiform tablet, U.7/80 (Figure 1), and in their paradigmatically amplified form as lines 30 to 57 (Figure 3), show how to change one tuning into the next in the opposite direction. In lines 30 to 57, a tuning is identified in the first line of each four-line segment (as in lines 1 to 28). In the second line, the *la-zaku* string-pair in the tuning identified in the first line is identified (again, as in lines 1 to 28), and added is the notion of touching (that is, plucking—perhaps "checking by ear") the strings of the *la-zaku* string-pair. (U.7/80 does not specify whether the strings are to be touched simultaneously or successively.) The third line identifies the string (or strings)

Figure 3. Lines 13 to 19 of Figure 1 (above) amplified paradigmatically to comprise 28 lines, numbered 30 to 57: see also Figures 1 and 2, above.

30. harp/lyre 62
31. touch [*la-zaku*] 25
32. alter? 2 and 9
33. harp/lyre 36
34. harp/lyre 36
35. t[ouch] *la-zaku* 62
36. alter? 6
37. [harp/lyre 73]
38. [harp/lyre 73]
39. [touch *la-zaku* 36]
40. [alter? 3]
41. [harp/lyre 47]
42. [harp/lyre 47]
43. [touch *la-zaku* 73]
44. [alter? 7]
45. [harp/lyre 14]
46. [harp/lyre 14]
47. [touch *la-zaku* 47]
48. [alter? 4]
49. [harp/lyre 51]
50. [harp/lyre 51]
51. [touch *la-zaku* 14]
52. [alter? 1 and 8]
53. [harp/lyre 25]
54. [harp/lyre 25]
55. [touch *la-zaku* 51]
56. [alter? 5]
57. [harp/lyre 62]

that is (or are) to be altered (yet again, as in lines 1 to 28). Most importantly, rather than specifying the string-pair that has been changed from *la zaku* in the second line by the string alteration of the third line, the fourth line identifies the number-pair of the tuning that results from this alteration. (Because lines 30 to 57 specify re-tunings in the opposite direction from the specifications in lines 1 to 28, the number-pair that results from the alteration of the third line is not the same as the number-pair that identifies the resulting tuning.)

As in lines 1 to 28, and consistent with the above interpretation of lines 1 to 28, *zaku* string-pairs accumulate cyclically, resulting in each of the tunings specified in lines 30 to

57. For instance, with regard to the 62 tuning indicated in lines 57 (and 30 and 0), line 31 specifies that the 25 string-pair is *la zaku*, whereas:

> line 30 implies that the 62 string-pair is *zaku* (as line 28 has specified),
> lines 33 and 34 specify that the 36 string-pair is *zaku* (as have lines 24 and 25),
> lines 37 and 38 specify that the 73 string-pair is *zaku* (as have lines 20 and 21),
> lines 41 and 42 specify that the 47 string-pair is *zaku* (as have lines 16 and 17),
> lines 45 and 46 specify that the 14 string-pair is *zaku* (as have lines 12 and13),
> and
> lines 49 and 50 specify that the 51 string-pair is *zaku* (as have lines 8 and 9).

As in lines 1 to 28, in each of the seven tunings specified in lines 30 to 57, 6 of the 7 3-mod-7 string-pairs are *zaku*, and the remaining 3-mod-7 string-pair is *la zaku*. Moreover, the cycle of re-tunings in lines 30 to 57 could be extended infinitely to include additional lines that could be numbered 58, 59, 60…. As well, the cycles that lines 1 to 28 and lines 30 to 57 comprise are continuous with each other, meeting as they do between lines 28 and 30, and forming as they do an ordering of re-tunings that could be modeled by a figure eight or by a one-dimensional circle that could be traversed both clockwise and counterclockwise.

MESOPOTAMIAN TUNING AS A 2-GAP CYCLE

As comprehensively formulated by Norman Carey (1998), a tuning or scale or, for that matter, any modular cycle is ‘well-formed’ (WF) and ‘non-degenerate’ if and only if all but one of the intervals that span a certain number of degrees or steps has the same specific magnitude. Accordingly, the seven tunings specified paradigmatically in U.7/80 are well-formed and non-degenerate, for all but one of the 3-mod-7 string-pairs that result from plucking the strings in each of the tunings is *zaku* and the remaining 3-mod-7 string-pair is *la zaku*.

By way of illustration, one can consider the specific magnitudes (to the nearest tenth of a cent) in the three cycles of Figure 4, where the specific magnitude of the modular interval is the ratio 2/1 = 1200 cents. Each of the cycles in Figure 4 is 'generated,' insofar as each tone is part of a cycle (or sub-cycle) of intervals whose specific size is the same (in these cases, ~514.3 or 500 cents).

In an ideal equiheptatonic cycle, the specific magnitude of the generating interval can be considered (3/7)*1200 cents ≈ 514.3 cents. The instances of the 3-mod-7 generating

Figure 4. Three tunings that illustrate 1-, 2- and 3-Gap cycles: note that the 1-Gap equiheptatonic cycle (a) could also be regarded as generated by the cycle 12, 23, 34, 45, 56, 67, (71) or by the cycle 13, 35, 57, 72, 24, 46, (61).

a) ideal equiheptatonic cycle (1-Gap): specific magnitude of the generating interval is (3/7)*1200 ≈ 514.3 cents (or its mod-1200-cent complement, ~685.7 cents)

degrees	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**(8=1)**
specific values (in cents)	0	171.4	342.9	514.3	685.7	857.1	1028.6	(1200=0)

generating intervals: 41, 15, 52, 26, 63, 37, 74

b) equally tempered diatonic cycle (2-Gap): specific magnitude of the generating interval is 500 cents (or its mod-1200-cent complement, 700 cents)

degrees	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**(8=1)**
specific values (in cents)	0	200	400	500	700	900	1100	(1200=0)

generating intervals: 41, 15, 52, 26, 63, 37

c) equally tempered hexatonic cycle (3-Gap): specific magnitude of the generating interval is 500 cents (or its mod-1200-cent complement, 700 cents)

degrees	**1**	**2**	**3**	**4**	**5**	**6**	**(7=1)**
specific values (in cents)	0	200	400	500	700	900	(1200=0)

generating intervals: 41, 15, 52, 26, 63 interval comprise the following degrees, relative to a modular interval that spans seven steps:

> 4 and 1, 1 and 5, 5 and 9 (=2 mod-7), 2 and 6, 6 and 10 (=3 mod-7), 3 and 7, and 7 and 4.

The specific magnitude of the generating interval between each of these pairs of degrees is ~514.3 cents: from 4 to 1, 514.3 - 0 = 514.3; from 1 to 5, 0 - 685.7 = (0+1200) - 685.7 = 514.3; from 5 to 2, 685.7 - 171.4 = 514.3; and so forth. Moreover, the specific magnitudes of the 1-step intervals are all the same, namely, ~171.4 cents: hence, the term '1-Gap.'

In an equally tempered diatonic cycle, the specific magnitude of the generating interval is (5/12)*1200 cents = 500 cents. As in an ideal equiheptatonic cycle, the instances of the generating interval comprise the following degrees, relative to a modular interval that spans seven steps:

> 4 and 1, 1 and 5, 5 and 9 (=2 mod-7), 2 and 6, 6 and 10 (=3 mod-7), and 3 and 7.

Whereas the specific magnitude of the generating interval between each of these pairs of tones in an ideal equiheptatonic cycle is ~514.3 cents, in an equally tempered diatonic cycle the corresponding interval is 500 cents: from 4 to 1, 500 - 0 = 500; from 1 and 5, 0 - 700 = (0+1200) - 700 = 500; from 5 to 2, (700 - 200) = 500; and so forth. However, in contrast to an ideal equiheptatonic cycle, the interval comprising degrees 7 and 4 in an equally tempered diatonic cycle differs in specific magnitude from the six other intervals that span 3-mod-7 steps: the specific magnitude of the interval from 4 to 7 is 500 - 1100 = (500+1200) - 1100 = 600 cents, rather than 500 cents. Further, among the 1-step intervals, there are 2 specific magnitudes. The specific magnitude of 12, 23, 45, 56, and

67 is 200 cents; the specific magnitude of 34 and 78 (i.e., 71 mod-7) is 100 cents: hence the term ‘2-Gap.’

As in an equally tempered diatonic cycle, the specific magnitude of the generating interval in an equally tempered hexatonic cycle is (5/12)*1200 cents = 500 cents. Unlike ideal equiheptatonic and equally tempered diatonic cycles, the instances of the generating interval comprise the following degrees, relative to a modular interval that spans six steps (rather than seven steps):

4 and 1, 1 and 5, 5 and 8 (=2 mod-6), 2 and 6, and 6 and 9 (=3 mod-6).

Unlike an ideal equiheptatonic cycle or an equally tempered diatonic cycle, in a hexatonic cycle there are only five instances of the generating interval, i.e., there is no counterpart to the interval comprising degrees 3 and 7 in an equally tempered diatonic cycle. As in an equally tempered diatonic cycle, the specific magnitude of the generating intervals comprised by these five pairs of tones is 500 cents: from 4 to 1, 500 - 0 = 500; from 1 to 5, 0 - 700 = (0+1200) - 700 = 500; from 5 to 8 (=2 mod-6), 700 - 200 = 500, from 2 to 6, 200 - 900 = (200+1200) - 900 = 500, and from 6 to 3, 900 - 400 = 500. Unlike both an equiheptatonic cycle and an equally tempered diatonic cycle, there are three specific magnitudes among the 1-step intervals. The specific magnitude of 12, 23, 45, and 56 is 200 cents; the specific magnitude of 34 is 100 cents; and the specific magnitude of 67 (i.e., 61 mod-6) is 300 cents: hence the term ‘3-Gap.’

According to the 3-Gap Theorem (Slater 1950, 528-34; Sós and Erdös 1957; Sós 1958; Swierczkowski 1958; Slater 1967; Chung and Graham 1976; Ravenstein 1988, 361-62)—also known as the 3-Difference or 3d Difference Theorem (Vijay 2008, 1655; Liang 1979, 325) and the [Hugo] Steinhaus Conjecture (Erdös 1940, 217) and also directly related to the Euclidean algorithm (or Euclid's algorithm) and Bjorklund's algorithm, which have been cited with regard to rhythmic cycles (Toussaint 2005, 47-49)—the three pitch cycles illustrated in Figure 4 exemplify the only three possibilities for a generated cycle.

Among these three possibilities, the ideal equiheptatonic and equally tempered diatonic cycles are 'well formed' (WF), for in both kinds of cycle every instance of the generating interval spans the same number of steps. However, in an equally tempered diatonic cycle one of the intervals that spans the same number of steps as the generating interval differs in specific magnitude from the generating interval and there are two specific magnitudes among the 1-step intervals. In contrast, all of the intervals in an ideal equiheptatonic cycle that span the same number of steps as the generating interval have the same specific magnitude and there is only one specific magnitude among the 1-step intervals. To convey these contrasts, Norman Carey and David Clampitt (1989, 200-02) have termed 1- and 2-Gap cycles, respectively, 'degenerate well-formed' and 'non-degenerate well-formed.' Rather than the terms 'degenerate well-formed' and 'non-degenerate well-formed,' the rest of this study employs the terms '1-Gap' and '2-Gap.'

As emphasized above, a significant contrast between 2-Gap cycles on one hand and both 1- and 3-Gap cycles on the other hand is that ***only in a 2-Gap cycle*** is it the case that ***there is an interval such that all but one of the intervals that span the same number of steps as this interval are the same in specific magnitude.*** In the equally tempered diatonic illustration, the specific magnitude of six of the seven 3-step intervals is the same, namely, 500 cents, from 4 to 1, 1 to 5, 5 to 2, 2 to 6, 6 to 3, and 3 to 7, and the specific magnitude of the remaining interval, from 7 to 4, is different, namely, 600 cents.

This feature is unique to 2-Gap cycles and is conveyed by U.7/80's paradigmatic arrangement of *zaku* and *la-zaku* intervals, for as shown above, the specific magnitude of six of the seven 3-mod-7 string-pairs in each of the seven Mesopotamian tunings is the same, namely, *zaku*, and the specific magnitude of the remaining 3-mod-7 string-pair is different, namely, *la zaku*.

12 Possible 7-Step 2-Gap Tunings

In general, if the generating interval spans d_g steps and the number of steps in the modular interval is d_m, the exceptional interval also spans $(d_g*d_m) – (d_g*(d_m – 1)) = d_g$ steps. In the case of 7-step 2-Gap tunings, there are three main possibilities:

a) the generating interval spans 1 step and the exceptional interval spans 1*7 – (1*6) = 1 step;
b) the generating interval spans 2 steps and the exceptional interval spans (2*7) – (2*6) = 2 steps;
c) the generating interval spans 3 steps and the exceptional interval spans (3*7) – (3*6) = 3 steps.

For each of these three main possibilities, there are two possibilities with regard to the specific magnitudes of the generating interval and the exceptional interval. The generating interval might be smaller than the exceptional interval, as in the instance of an equally tempered diatonic cycle, where the specific magnitudes of the generating interval and the exceptional interval are, respectively, 500 and 600 cents. Alternatively, the generating interval might be larger than the exceptional interval, as in the instance of seven-tone *pélog*, where, as indicated below, the specific magnitudes of the generating and exceptional intervals are arguably $(4/9)*s_g$ and $(3/9)*s_g$, i.e., $s_g/3$: if the specific magnitude of the modulus is 1200 cents, these values are, respectively, ~533 and 400 cents.

By way of further illustration, if d_m, the number of steps the modular interval spans, is 7, six groups of values are possible for a 2-Gap cycle's generating interval's number of steps (d_g) and specific magnitude (s_g), relative to the exceptional interval, as specified in Figure 5. For each of the six possibilities, the specific magnitude of the generating interval lies within a range of values rather than necessarily having only a single value. Moreover, 30% of the modular interval's range of specific magnitudes is excluded as possible for the specific magnitude of a 7-step 2-Gap cycle's generating interval.

For example, as Figure 5 shows, if the specific magnitude of the modular interval is 1200 cents, the generating interval's specific magnitude cannot be between 200 and 300 cents, between 400 and 480 cents, between 720 and 800 cents, or between 900 and 1000

Figure 5. 6 possibilities for s_g, the specific magnitude of the generating interval, relative to the specific magnitude of the modular interval, s_m, depending on the number of steps spanned by the generating interval, d_g: for the illustrative numerical values, s_m = 1200 cents.

d_g	**s_g:**	
	smaller than the exceptional interval	**larger than the exceptional interval**
1	$(0/3)*s_m < s_g < (1/7)*s_m$ $0 < s_g <$ ~171.4 cents	$1/7)*s_m < s_g < (1/6)*s_m$ ~171.4 $< s_g <$ 200 cents
2	$(1/4)*s_m < s_g < (2/7)*s_m$ $300 < s_g <$ ~342.9 cents	$(2/7)*s_m < s_g < (1/3)*s_m$ ~342.9 $< s_g <$ 400 cents
3	$(2/5)*s_m < s_g < (3/7)*s_m$ $480 < s_g <$ ~514.3 cents	$(3/7)*s_m < s_g < (1/2)*s_m$ ~514.3 $< s_g <$ 600 cents

cents (each of these exclusive), that is, 100+80+80+100 = 360 cents are impossible as compared to a modular interval of 1200 cents.

For each of the 3*2 = 6 possibilities illustrated in Figure 5, there are two possible directions in which an interval that spans 1, 2, or 3 steps might generate a 7-step 2-Gap cycle. For instance, a 3-step interval might generate a diatonic cycle from pitch class B in either of the following directions:

upward: B E A D G C F
downward: B F♯ C♯ G D A E

Depending on whether they are generated upward or downward from B, these diatonic cycles have the following successions of small (s) and large (L) 1-step intervals:

generated upward from B by a 3-step interval:

E		D		C		B		A		G		F		(E)
	L		L		s		L		L		L		s	

generated downward from B by a 3-step interval:

E♯		D♯		C♯		B		A♯		G♯		F♯		(E♯)
	L		L		L		s		L		L		s	

A similar situation holds for 7-step 2-Gap cycles generated by a 1- or 2-step interval. Accordingly, there are 3*2*2 = 12 general ways in which a 7-step 2-Gap cycle might be generated. Figure 6 illustrates the twelve possibilities for the *nitkibli* tuning, in which plucking string-pair 47 results in the exceptional interval.

GENERIC-SPECIFIC SAMENESS IN 2-GAP CYCLES

Discerning the mod-7 2-Gap structure of Mesopotamian tuning not only narrows the possibilities of the Hurrian pieces' degree-based structure to twelve main kinds of step-

Figure 6. Twelve possible *nitkibli* tunings, where the specific magnitude of the modular interval, s_m, is 2/1 = 1200 cents, the number of steps in the modular interval, d_m, is 7, the number of steps in the generating interval, d_g, is 1, 2, or 3, the magnitude of the generating interval is small (s) or large (L), the direction of generation is upward or downward from string 4 to string 1, and the fundamental frequency of string 1 is 0 cents.

d_g	s_g	direction	strings:						
			1	**2**	**3**	**4**	**5**	**6**	**7**
1	s=150	upward	0	300	600	-150	150	450	750
1	s=150	downward	0	-300	-600	150	-150	-450	-750
1	L=190	upward	0	380	760	-190	190	570	950
1	L=190	downward	0	-380	-760	190	-190	-570	-950
2	s=320	upward	0	640	1280	-320	320	960	1600
2	s=320	downward	0	-640	-1280	320	-320	-960	-1600
2	L=370	upward	0	740	1480	-370	370	1110	1850
2	L=370	downward	0	-740	-1480	370	-370	-1110	-1850
3	s=500	upward	0	1000	2000	-500	500	1500	2500
3	s=500	downward	0	-1000	-2000	500	-500	-1500	-2500
3	L≈533	upward	0	1067	2133	-533	533	1600	2667
3	L≈533	downward	0	-1067	-2133	533	-533	-1600	-2667

cycle; as well, the mod-7 2-Gap structure introduces distinctions between particular instances of generic intervals. Each second, third, fourth, etc. is not only generically the same as, every other second, third, fourth, etc. (respectively), but also, the large seconds are more similar to one another than they are to the other, small seconds by virtue of being the same not only generically but also with regard to their specific magnitudes, and similarly for each of the other kinds of generic interval.

A consequence of Mesopotamian tuning's 2-Gap structure is that it maximizes generic-specific sameness. Briefly, among all cycles whose modular interval spans a certain number of steps, d_m, and in which any interval that spans a particular number of steps differs in specific magnitude from at least one interval that spans the same number of steps, a 2-Gap cycle comprises the greatest number of pairs of intervals that are the same both generically and specifically.

For instance, in the equally tempered diatonic cycle E D C B A G F (E), the number of intervals that span three steps and whose specific magnitude is 500 cents is 6 (i.e., the perfect fourths BE, EA, AD, DG, GC, and CF) and the number of fourths that span three steps and whose specific magnitude is 600 cents is 1 (i.e., the augmented fourth FB). Among the six perfect fourths, there are $C(6,2) = 6*(6-1)/2 = 6*5/2 = 15$ pairs of intervals that are generically and specifically the same (namely, BE and EA, BE and AD, BE and DG, …, and GC and CF), whereas the single augmented fourth (FB) is not a part of any pair of intervals; that is, the number of sameness-pairs among the augmented fourths is zero: $C(1,2) = 1*0/2 = 0$. Figure 7 specifies the number of instances of each generic-specific interval in such an equally tempered diatonic cycle and the number of generic-specific sameness-pairs that result from each.

As Figure 7 shows, the number of generic-specific sameness-pairs differs among the various generic-specific intervals in an equally tempered diatonic cycle. There are more sameness-pairs among the perfect fourths and perfect fifths (namely, fifteen of each) than among any of the other generic-specific intervals; among these other generic-specific intervals, there are more sameness-pairs among the major seconds and minor sevenths, and so forth.

In general, the numbers of instances of particular generic-specific intervals that span 1, 2, or 3 steps in a 7-step 2-Gap cycle are 1, 2, 3, 4, 5, and 6; similarly, the numbers of instances of particular generic-specific intervals that span 6, 5, or 4 steps in such a cycle are 1, 2, 3, 4, 5, and 6. More generally, if the modular interval spans d_m steps, a 2-Gap

Figure 7. Numbers of generic-specific sameness-pairs among intervals in an equally tempered diatonic cycle that span 1 to 6 steps and whose specific magnitudes (specified in cents) are small (s) or large (L).

number of steps:	**1**		**2**		**3**		**4**		**5**		**6**	
specific magnitude:	100	200	300	400	500	600	600	700	800	900	1000	1100
	s	L	s	L	s	L	s	L	s	L	s	L
number of intervals:	2	5	4	3	6	1	1	6	3	4	5	2
number of generic-specific sameness pairs:	1	10	6	3	15	0	0	15	3	6	10	1

cycle's 'generic-specific vector' comprises the following values: 1, 2, 3, …, $d_m - 2$, $d_m - 1$; $d_m - 1$, $d_m - 2$, …, 3, 2, 1. Accordingly, the number of sameness-pairs in a 7-step 2-Gap cycle is as follows:

$$[C(1,2)+C(2,2)+C(3,2)+C(4,2)+C(5,2)+C(6,2)] \\ +[C(6,2)+C(5,2)+C(4,2)+C(3,2)+C(2,2)+C(1,2)]$$

$$= (0+1+3+6+10+15)+(15+10+6+3+1+0)$$

$$= 70.$$

More generally, the number of sameness-pairs in a 2-Gap cycle is the following tetrahedral (or triangular pyramidal) number (cf. AT&T Labs Research 2010):

$$2*C(d_m,3) \\ = (d_m)*(d_m - 1)*(d_m - 2)/3.$$

In any of the infinite number of possible mod-d_m cycles that are not 1-Gap cycles, the maximum number of generic-specific sameness pairs, namely, $2*C(d_m, 3)$, occurs in a 2-Gap cycle. In a 1-Gap cycle the number of generic-specific pairs is as follows:

$$(d_m-1)*(d_m*(d_m-1))/2$$

$$= (d_m)*((d_m - 1)\text{^}2)/4,$$

or if one counts unisons and octaves,

$$(d_m+1)*(d_m*(d_m-1)/2)$$

$$= (d_m + 1)*(d_m)*(d_m - 1)/2).$$

In any event, ***among all possible d_m-step tunings, except an 'equitonic,' 1-Gap tuning, generic-specific sameness is maximized in a 2-Gap tuning***.

In further contrast, a 3-Gap cycle yields fewer generic-specific sameness-pairs than a 2-Gap cycle. By way of illustration, Figure 8 specifies the generic-specific interval vector for the equally tempered hexatonic cycle C D E F G A. In Figure 8 the generic-specific intervals CD, DE, EF, FG, and GA correspond to their counterparts in the equally tempered diatonic cycle C D E F G A B: in each case, these intervals span one step and their specific magnitudes are 100 cents or 200 cents. However, whereas AC's specific magnitude is the same in both kinds of cycle, namely, 300 cents, AC spans only one step in an equally tempered hexatonic cycle rather than the two steps that AC spans in an equally tempered diatonic cycle. Similarly, in C D E F G A, GC and AD span two steps rather than three; FC, GD, and AE span three steps rather than four, and so forth. In sum, as Figure 8 shows, there are only 24 generic-specific sameness-pairs if C D E F G A is construed as a hexatonic cycle.

In contrast, if C D E F G A is construed as part of an equally tempered diatonic cycle, the intervals CD, DE, EF, FG, and GA span one step as in an equally tempered hexatonic cycle, but AC spans two steps as do CE, DF, EG, and FA, and so forth, for a total of 40 generic-specific sameness-pairs, rather than 24. Interval by interval, then, an equally tempered diatonic construal results in more generic-specific sameness-pairs than an equally tempered hexatonic construal.

Figure 8. Generic-specific vectors and numbers of generic-specific sameness-pairs if C D E F G A is construed a) as an equally tempered hexatonic cycle in which there are 6 steps, and b) as part of an equally tempered diatonic cycle in which there are 7 steps.

a) hexatonic

number of steps:	**1**			**2**			**3**		**4**			**5**		
specific magnitude:	100	200	300	300	400	500	500	700	700	800	900	900	1000	1100
number of intervals:	1	4	1	2	2	2	3	3	2	2	2	1	4	1
number of generic-specific sameness-pairs:	0	6	0	1	1	1	3	3	1	1	1	0	6	0

b) diatonic

number of steps:	**1**		**2**		**3**	**4**	**5**		**6**	
specific magnitude:	100	200	300	400	500	700	800	900	1000	1100
number of intervals:	1	4	3	2	5	5	2	3	4	1
number of generic-specific sameness-pairs:	0	6	3	1	10	10	1	3	6	0

On the whole, and more generally, a 2-Gap construal results in more generic-specific sameness-pairs than a 3-Gap counterpart, for, as discussed further below, the generic categories of a 2-Gap construal comprise at least as many or more intervals: e.g., three 2-step intervals of 300 cents in an equally tempered diatonic construal result in 3*2/2 = 3 generic-specific sameness-pairs, whereas one 1-step interval and two 2-step intervals of 300 cents in an equally tempered hexatonic construal result in (1*0/2)+(2*1/2) = 0+1 = 1 sameness-pair. (Cf. a similar situation with regard to so-called ‘ambiguities’ and 'contradictions' in Rahn 1991, 44-49). Accordingly, ***among all possible generated tunings, except a 1-Gap, equitonic tuning, generic-specific sameness is maximized directly in a 2-Gap cycle or in a 3-Gap cycle that is construed in terms of its corresponding 2-Gap cycle***.

2-Gap Cycles In Subsequent Traditions

2-Gap cycles have been quite widespread among temporally and geographically far-flung musical traditions (Figure 9). Like the 7-step 2-Gap tuning of Mesopotamia, Aristoxenus's diatonic tuning (ca. 325 BCE: Barker 1989, 141-44) was produced 'by ear,' or as Aristoxenus says, 'by the intrinsic nature of melody itself' (Barker 1989, 139,

Figure 9. Ideal, numerical formulations of 2-Gap cycles: to facilitate comparison, the modular interval's specific magnitude is 1200 cents for each cycle; 7-step cycles are generated upward from degree 4 by a 3-step generating interval; 5-step cycles are generated upward from degree 3 by a 2-step generating interval; all values are to the nearest cent.

7-step 2-Gap cycles:

degrees	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**(8=1)**
a) diatonic:								
i. Pythagorean	0	204	408	498	702	906	1110	(1200=0)
ii. ¼-comma	0	193	386	503	697	890	1083	(1200=0)
iii. 12-ET	0	200	400	500	700	900	1100	(1200=0)
b) 7-step *pélog*	0	133	267	533	667	800	933	(1200=0)

5-step 2-Gap cycles:

degrees	**1**	**2**	**3**	**4**	**5**	**(6=1)**
a) anhemitonic pentatonic	0	204	498	702	906	(1200=0)
b) 5-step *pélog*	0	133	533	667	800	(1200=0)
c) 5-step equiheptatonic	0	171	514	686	857	(1200=0)

160), rather than by means of the Pythagorean tradition's numerical formulation of string-lengths, e.g., by Philolaus (ca. 425 BCE: Huffman 1993, 145-47). Indeed, notwithstanding more than two millennia of numerical formulations, ancient Greek and historically related western European diatonic tunings have been realized by ear. (Cf. in this regard Cristiano M. L. Forster's (2010, 3.17) distinction between 'theoreticians' and 'practicing musicians'.)

In conjunction with a 2/1 ratio for the octave, numerical formulations of the generating interval's specific magnitude for diatonic cycles have included i) the ratios 4/3 and 3/2, respectively ~498 and ~702 cents, for so-called 'Pythagorean' tuning from Greek antiquity to the European Renaissance; ii) 2/(5^(1/4)) and (5^(1/4))/1, respectively ~503 and ~697 cents, for quarter-comma mean-tone tuning (Figure 9.a.ii); and iii) 2^(5/12)/1 and 2^(7/12)/1, respectively 500 and 700 cents, for twelve-semitone equal temperament (Figure 9.a). However, long before they were formulated numerically, both quarter-comma mean-tone and twelve-semitone equal temperament were specified in perceptual, phenomenal terms that are so vague as not to distinguish one from the other. According to Franchino Gaffurio (1496), perfect fifths were to be made smaller 'by a very small and hidden and somewhat uncertain quantity,' and according to Pietro Aron (1523-29), major thirds were to be tuned 'sonorous and just' and 'as united as possible' (Lindley 2009). In both cases, the specific magnitudes of perfect fifths and major thirds would be smaller than in received Pythagorean tuning and these features are characteristic of both quarter-comma mean-tone and twelve-semitone equal temperament. More importantly, Pythagorean, quarter-comma, equally tempered, and other ways of realizing diatonic are, to employ Easley Blackwood's (1985) term, 'recognizable' as diatonic tunings. According to the present formulation, these tunings are recognizable as diatonic insofar as six of the seven fourths are perceptibly smaller than the remaining fourth, five of the seconds are perceptibly larger than the other two seconds, and four of the thirds are perceptibly smaller than the other three thirds.

To be sure, such recognizable diatonic tunings have been formulated numerically in a great variety of ways in order to realize particular results. For example, reducing the specific magnitudes of the perfect fifths results in simultaneously sounding major thirds whose spectra create slower rates of beating between the fifth and fourth partials of the lower and upper tones and also increases what Steven Block and Jack Douthett (1994, 31-40) have characterized as 'evenness' throughout the entire cycle (cf. also Clough and Douthett 1991 and Demaine et al. 2009, 441-47). Nonetheless, in such formulations, perfect fifths and major thirds remain recognizably larger than, respectively, diminished fifths and minor thirds and similarly for the minor and major seconds, so that each can be considered an instance of what might be termed 'general diatonic.' Similarly, the manifold ways in which diatonic has been realized in actual practice can be regarded as further instances of general diatonic.

For several centuries, 7-step *pélog* tunings of Java and Bali were produced by ear, i.e., without numerical formulation. The most telling respect in which Indonesian *pélog* tunings differ from their ancient Greek and western European diatonic counterparts can be found in the specific magnitudes of their 3-step generating intervals being larger than the exceptional interval, e.g., respectively, ~533 cents *vs* ~400 cents, in contrast to ~500 cents *vs* ~600 cents in diatonic (Kunst 1973, 55-57; Rahn 1978, 71-76; Braun 2002). Granted, *pélog* tunings seem to have manifest rather great variation around such ideal values as ~533 cents and such divergence has involved not only intervals of one to six steps but also the octave and unison (cf., e.g., Hood 1966, 33, 42, 44-47; Surjodiningrat et al. 1972, 17 39, 43, 47, 53; Cartertette and Kendall 1994, 59-64). All the same, such variation has not prevented *pélog* tunings from being recognizable as such, i.e., as *pélog* tunings. As in the case of European diatonic tuning, nuances within a more general 3-step 2-Gap framework have been a product of expressive concerns on the part of tuners (cf., e.g., Dowling and Harwood 1986, 101-04, 120) without disrupting the relationships between smaller and larger versions of the 1-, 2-, and 3-step intervals.

Additionally, the 5-step 2-Gap cycle of anhemitonic pentatonic (e.g., F G A C D, which can be generated as A D G C F) was formulated numerically in ancient China, e.g., in *Guanzi*, ca. 225 BCE: Rickett 1998, vol. 2, 259, 263) with 4/3 and 3/2 as the generating interval's complementary specific magnitudes. Like diatonic and 7-degree *pélog*, anhemitonic pentatonic has been a product of phenomenal, perceptual tuning in several musical settings, as have the 5-degree version of Indonesian *pélog* and, in Thai court music, the 5-step version of equiheptatonic that has been employed in relatively 'simpler' styles (Morton 1968, 9-10; 1976, 32-33).

Without insisting on particular historical connections between Mesopotamian tuning and musical practices that have been documented much later in other cultures, one can observe that 2-Gap cycles have flourished in several settings, both with and without the adjunct of precise numerical formulations. In this regard, relative cognitive simplicity can be assessed by the number of pairs of intervals that are perceptibly smaller and larger versions of intervals that span the same numbers of steps, and rather than or in conjunction with historical influence, relative cognitive simplicity can serve as a basis for explaining such recurrence and persistence (e.g., as instances of polygenesis, where generic-specific sameness would be a widespread reinforcer).

Generic-Specific Sameness In H.6

Turning from 2-Gap cycles in general to the concrete details of particular intervals employed in h.6, one can note that if each degree, i.e., degree-class, were realized exactly once in a piece or passage, the number of generic-specific sameness-pairs specified above, namely $2*C(d_m,3)$, would be realized precisely. However, if each degree-class were realized twice, the frequencies of generic-specific sameness-pairs would increase. As shown in Figure 10, if each degree-class were realized only once, there would be only one instance of the pair of degree-classes that constitute the exceptional interval, which in h.6 is the *la-zaku* interval 47; accordingly, if each degree-class were realized only once, C(1,2) = 0 instances of the *la-zaku* interval would be the same. If each degree-class were realized twice, the two degree-classes that constitute the *la-zaku* interval, namely, the pair of degree-classes produced by strings 4 and 7 in *nitkibli* tuning, would each be realized two times, so that there would be 2*2 = 4 instances of the *la-zaku* interval among which there would be C(4,2) = 6 pairs of the *la-zaku* interval that would be generically and specifically the same.

Figure 10. Numbers of generic-specific sameness-pairs if each degree-class in a 7-step 2-Gap cycle is realized once or twice (cf. Figure 7).

each degree-class realized …	**no. of generic-specific sameness-pairs among intervals of 1, 2, and 3 steps whose ideal frequencies are…**					
	1	**2**	**3**	**4**	**5**	**6**
once:	0	1	3	6	10	15
twice:	6	28	66	120	190	272

In general, the number of pairs of intervals that are generically and specifically the same is ((i^2)*v)*(((i^2)*v)-1)/2, where i is the number of instances of both degrees of each interval of a particular generic-specific kind and v is the number of intervals of this kind in the tuning's generic-specific vector.

The sixty-eight instances of particular degree-classes in h.6 cannot be realized perfectly evenly, because 68/7 ≈ 9.71, which is not a natural counting number. All the same, one can calculate the results of the closest approximation to such a perfectly even distribution by taking 68/7 as the ideal frequency of each degree-class. In Figure 11, the six kinds of generic-specific interval are specified according to their relative magnitudes in 3-mod-7 2-Gap tunings (i.e., general diatonic and general *pélog*) and the numbers of sameness-pairs that would result from a perfectly even distribution are compared with the actual numbers of sameness-pairs in h.6.

Comparing the ideal values of Figure 11(a) with the actual numbers of sameness-pairs in h.6 (Figure 11(b)), one finds that ***the generic-specific sameness-pairs in h.6 that exceed the ideal frequencies correspond to*** the most frequent generic-specific intervals in Figure 10 and the first three generic-specific intervals in Figure 11, namely, the small fourth, large second, and small third in a diatonic tuning or the large fourth, small second, and large third in a *pélog* tuning, or their counterparts in the other ten possible kinds of 2-Gap tuning, i.e., ***the kinds of sameness pairs that are most frequent in a perfectly even distribution.*** Conversely, those that are under-realized in h.6 correspond to the generic-specific intervals that are least frequent in the general, somewhat abstract cycle. Further, the total number of actual sameness-pairs exceeds the total in a perfectly even distribution of degree-classes. That there are actually ***more sameness-pairs than in an***

Figure 11. Numbers of generic-specific sameness-pairs in:
a) a perfectly even distribution of particular degrees in h.6 according to a 3-mod-7 2-Gap tuning of the diatonic or *pélog* variety, compared with
b) the numbers of generic-specific sameness-pairs that actually occur among the 68 instances of particular degrees in h.6.
Cf. Figure 3 in the first part of this study and Figure 13, below. String-pairs (e.g., 14) exemplify the corresponding generic-specific intervals in *nitkibli* tuning, and the numbers of sameness-pairs in column *a* are rounded to the nearest whole number.

generic-specific interval in: diatonic	***pélog***	**illustrative string-pair in *nitkibli* tuning**	**no. of sameness pairs in: a) a perfectly even distribution**	**b) h.6**
small 4th	large 4th	14	160,010	172,578
large 2nd	small 2nd	45	111,079	141,778
small 3rd	large 3rd	24	71,053	71,253
large 3rd	small 3rd	13	39,932	34,716
small 2nd	large 2nd	34	17,716	14,028
large 4th	small 4th	47	4,405	1,770
totals:			404,196	436,123

ideally even distribution of degree-classes indicates a ***tendency toward sameness rather than difference in h.6***. As well, the source of ***the greater frequencies of sameness- pairs among the kinds of sameness-pairs that are most frequent in a perfectly even distribution can be traced to the greater frequencies of individual degree-classes***.

Sameness Among Individual Degree-Classes In H.6

In any non-1-Gap cycle, individual degrees differ in their 'panoramas,' i.e., the intervals they form with other degrees (Clough et al. 1999, 102; see also Rahn 1996, 79-80 on 'maximal individuation'). As Figure 12 shows, the degree that corresponds to string 2 (which would be D in the white-key collection B C D E F G A that would be generated upward by a small fourth starting at B) is part of two small fourths (AD and DG), two large seconds (CD and DE), and two small thirds (BD and DF).

Figure 12. Number of sameness-pairs among generic-specific intervals that include particular strings, i.e., degree-classes, in a *nitkibli* tuning whose generating interval is a small fourth: for illustrative purposes, individual strings are identified with degree-classes in the white-key diatonic cycle.

string number:	**4**	**1**	**5**	**2**	**6**	**3**	**7**
letter-name in white-key collection:	**B**	**E**	**A**	**D**	**G**	**C**	**F**
interval in cycle with small-4th generating interval:							
small 4th	1	2	2	2	2	2	1
large 2nd	1	1	2	2	2	1	1
small 3rd	1	1	1	2	1	1	1
large 3rd	1	1	1	0	1	1	1
small 2nd	1	1	0	0	0	1	1
large 4th	1	0	0	0	0	0	1

More instances of string 2 would result in more sameness-pairs consisting of, e.g., small fourths, large seconds, and small thirds in a general diatonic tuning, as is the tendency in h.6 (Figure 11, above), and no additional sameness-pairs consisting of large thirds, small seconds, or large fourths. In contrast, B and F are part of one instance of each kind of interval: e.g., B is part of one small fourth (BE), one large second (AB), one small third (BD), one large third (GE), one small second (BC), and one large fourth (FB). Within h.6's tuning, B and F, the end-points of the *nitkibli* tuning cycle, are 'neutral' with regard to sameness-pairs, whereas D is 'biased' toward small fourths, large seconds, and small thirds, as is the cycle as a whole. This bias declines symmetrically from D through i) A and G and ii) E and C to iii) B and F.

In h.6, relatively large frequencies of D and A and G, and relatively small frequencies of E and C and B correspond to this polarity (Figure 13). Within this overall pattern, the only exception is the large frequency of string 7 (F in Figures 12 and 13). As discussed in the first part of this study, string 7 is the site of an anomaly just after the outset of h.6; as

Figure 13. Frequencies of individual string-numbers in h. 6: whereas the chi-squared probability is .34 > .05 (cf. Figure 3 in the first part of this study and Figure 11, above), strings 2, 5, 6, and 7 exceed the expected frequencies, as indicated by asterisks; as in Figure 12, above, individual strings are identified with degree-classes in the white-key collection for illustrative purposes.

string-numbers:	**1**	***2**	**3**	**4**	***5**	***6**	***7**
letter-names in white-key collection:	**E**	***D**	**C**	**B**	***A**	***G**	***F**
frequencies:							
actual	8	*12	6	5	*13	*12	*12
expected	9.71	9.71	9.71	9.71	9.71	9.71	9.71

well, within h.6 string 7 is part of all 6 of the 2-mod-7 string-pairs that immediately precede a 3-mod-7 string-pair, each of which is an instance of the generating fourth (one-way chi-squared probability = .00 < .05). Moreover, that ***the frequencies of particular degree-classes correspond to the frequencies of sameness-pairs*** holds not only for generic-specific intervals but also for generic intervals and instances of the modular interval, which would include instances of unisons and octaves in an actual realization.

Sameness Among Instances Of The Modular Interval In H.6

In general, if a group of things is the same in some respect, it comprises more sameness-pairs than if one or more things in the group differ in that respect. For example, among n things that are all the same in a particular respect, there are (n)*(n-1)/2 = (n)*((n-1)/2) sameness-pairs; if one of these things differs in that respect, there are ((n-1)*(n-2)/2) + ((1)*(0)/2)) = (n-1)*(n-2)/2 = (n-2)*((n-1)/2) sameness-pairs, which are fewer than (n)*(n-1)/2 since n > n-2. More generally, if a group of things is partitioned more evenly, its number of sameness-pairs is smaller. If 2*n things are partitioned evenly into n things and n things, there are, in total, ((n)*(n-1)/2) + ((n)*(n-1)/2) = (n)*(n-1) =

n^2 - n sameness-pairs. If partitioned into n+1 and n-1 things, there are ((n+1)*(n)/2) + ((n)*(n-1))/2 = n^2 > n^2 - n sameness-pairs.

In general, a maximally uniform partitioning of a group into p parts results in the minimum number of sameness pairs among the p parts. As a consequence, if the frequencies of particular degree-classes are distributed unevenly, the number of modular sameness-pairs is greater than if they were distributed perfectly evenly. For instance, the number of modular sameness-pairs in h.6 is 53,956, which is larger than the number there would be if the frequencies of particular degree-classes were distributed perfectly evenly, namely, 43,745 (rounded to the nearest whole number).

Such an ***excess of unison sameness-pairs*** corresponds to the notions that tones which recur relatively often a) are relatively privileged in a mode or scale (e.g., Winnington-Ingram 1936, 2; Nettl 1964, 145-47, 157-60), b) result in greater redundancy in an information-theoretical analysis of musical style (e.g., Youngblood 1958, 29-35; cf., however, Cohen 1962, 155-59), and c) are to be accounted for in a preliminary, ‘pre-network input’ stage in an artificial-intelligence modeling of modal practice (e.g., Taskin 2005, 43-46). Moreover, ***as in the case of non-modular sameness- pairs*** considered above, ***the excessive number of modular sameness- pairs in h.6 can be traced, for the most part, to the string-degree-classes 2, 5, and 6*** (e.g., D, A, and G) ***and is consistent with a tendency to simplicity***.

Since a group of things that are the same in some respect comprises more sameness-pairs than if one or more things in the group differ, there are more generic sameness-pairs than generic-specific sameness-pairs in h.6 (respectively, 632,200 *vs.* 429,970), for generic-specific intervals are subsets of their generic counterparts. Nonetheless, as discussed above concerning sameness relationships in general, intervals that are generically and specifically the same are the same to a greater extent than intervals that are merely generically the same. In this sense, a 2-Gap cycle could be considered to result in sameness-pairs to a greater extent than its 1-Gap counterpart.

As each of the twelve fourths in h. 6 is an instance of the generating interval, the number of generic-specific sameness-pairs among the 3-mod-7 string-pairs of h.6, namely, C(12,2) = 66, is maximal. Although the differences between the actual and expected frequencies of the two kinds of 2-mod-7 string-pairs are not significant (Figure 14), both differences point again towards sameness rather than difference, for instances of the generating interval are more frequent than they would be in a perfectly even distribution, as are instances of what would be the small third in general diatonic (or the large third in general *pélog*, etc.).

REGISTRAL ASPECTS OF MESOPOTAMIAN TUNING

The preceding discussion has been framed in terms of modular classes: in particular, modular classes of seven string-numbers in the first part of this study and modular classes

Figure 14. Frequencies of generic-specific intervals among the 34 string-pairs of h.6: cf. Figures 5 and 7 in the first part of this study. For the purpose of illustration, generic-specific intervals are labeled according to their specific magnitudes in a mod-7 2-Gap cycle whose generating interval is a small fourth (i.e., general diatonic).

frequencies:	**3rd s:**		**4th s:**	
	small	**large**	**small**	**large**
actual	14	8	12	0
expected	12.6	9.4	10.3	1.7

of seven degrees in the present installment. In such a highly general formulation, the particular register of a particular degree and the distinction between higher and lower degrees are of no structural consequence. Nonetheless, determining whether a particular string on the nine-string harp or lyre described in U.3011 sounded higher or lower than another string has been a topic of considerable disagreement in studies of Mesopotamian music (e.g., Crocker 1997, 192-93 *vs.* Gurney and West 1998, 223-24).

'Smaller' And 'Larger' Strings

Early studies of Mesopotamian music concluded that the ordering of the fundamental frequencies produced by the first to the ninth strings that U.3011 lists for a nine-string lyre or harp was stepwise from lowest to highest (e.g., Duchesne-Guillemin 1963, 9; Duchesne-Guillemin 1966, 152; Güterbock 1970, 45; Wulstan 1971, 367, 378-80; Kilmer 1974, 72; Kilmer et al. 1976, 7-17). Bases for this conclusion were that a) the terms for the first and ninth strings of a nine-string harp or lyre listed in U.3011 are, respectively, 'fore' or 'front' (Akkadian: *qudmum*) and 'behind' or 'back' (Sumerian: *agagulla*; Akkadian: *úḫrum*), and b) in Mesopotamian pictorial representations of harps, the strings are successively longer from the string closest to the performer to the string farthest from the performer. However, as Raoul Vitale (1982, 243) subsequently pointed out, the front and back of a harp or lyre might have been considered to be 'the front' and 'the back' relative to the performer or relative to a listener facing the performer.

'Thin' And 'Small' Strings

Also favoring the conclusion that lower numbered strings produced lower fundamental frequencies, Wulstan (1968, 226-28) drew attention to a 'nick' at the top of one of the longest strings on most of the instruments in a pictorial representation of a harp ensemble ca. 650 BCE. Wulstan regarded these notches as a basis for explaining two of the string names in U.3011. In U.3011, the names of the third and fourth strings have generally been translated as, respectively, 'third …thin' (Sumerian: 3…*sig*; Akkadian: 3…*qatnu*) and 'fourth…small' (Sumerian: 4…*tur*). Wulstan reasoned that these strings served to coordinate the fundamental frequencies of the instruments in somewhat the same way as red and blue strings identify pitch classes C and F on modern European harps.

Relative to the pattern of successively shorter strings from front to back, the string named 'third thin' in U.3011 would, according to Wulstan, be about as long as the second string and hence would require a nick in the instrument's neck to accommodate the greater length in order for its fundamental frequency to be between the fundamental frequencies of the second and fourth strings listed in U.3011. In comparison with this longer, relatively thin string, the fourth string would appear discrepantly 'small,' i.e., short.

As Wulstan acknowledged, Mesopotamian pictorial representations would not convey clearly the relative thickness of the strings, which would be of relevance to the fundamental frequencies produced by harps and, especially, lyres. More important, details that could be considered to depict nicks are not evident in other Mesopotamian representations of harps (Rashid 1984, 34-153). Further, the fundamental frequency of a string is determined not only by its length and thickness, i.e., mass, but also by its tension.

Rather than putative nicks or the relative positions of the front and back of a harp or lyre, Vitale considered the terms ‘thin’ and ‘small’ for the third and fourth strings, on their own, as decisive for interpreting the registral ordering of the fundamental frequencies of a harp or lyre. In this regard, the connotations of the terms employed in naming the third and fourth strings are potentially relevant.

Connotations Of ‘Thin’ And ‘Small’

The Sumerian term *sa*, generally rendered as ‘string’ in translations of Mesopotamian documents relevant to tuning, also denoted, beyond musical contexts, ‘gut,’ ‘sinew,’ ‘tendon,’ ‘intestine(s),’ and ‘catgut string’ (Tinney 2006). The non-numerical Akkadian term for the third string, namely, *qatnu*, generally rendered as ‘thin’ in translations of Mesopotamian tuning documents, was also used to describe intestines and textiles. As well, *qatnu* was employed to contrast a narrow wall and a strong wall (*danna*: strong, thick, wide), a younger and an older sibling (*mahru*), and a soft and a loud voice (*kabar*: thick, strong, fat, sonorous). The non-numerical Sumerian term for the fourth string, *tur*, generally rendered as ‘small’ in translations of Mesopotamian documents germane to tuning, has also been translated as ‘young’ in non-musical contexts (Gelb et al. 1956-2006).

These non-numerical terms can be considered to have both acoustical and perceptual implications. The word *qatnu* would describe a string whose relatively small mass would result in a relatively high fundamental frequency; similarly, *tur* would describe a string whose relatively small mass and/or relatively short length would result in a relatively high fundamental frequency. Moreover, the use of 'intestine(s),' 'gut,' and 'catgut' for strings in general is consistent with the use of 'thin' to describe 'intestines,' and of the three main variables that affect the fundamental frequency of a string, only tension appears not to have been directly associated with the terms 'thin' and 'small.'

Perceptually, the use of 'small' for 'young' and the use of 'thin' for a younger sibling suggest correspondences between age on one hand and fundamental frequency on the other. Physiologically, the vocal cords of children and women (and hence young women) tend to be shorter than those of adult and adolescent males (whose voices have changed) and their voices tend to have higher fundamental frequencies (Gordon 1978, 116-17). Moreover, beyond Mesopotamian musical culture, instrumental tones with higher fundamental frequencies have been termed 'small,' e.g., by the Venda of South Africa (Blacking 1970, 12, 18), and have been associated with younger persons. For instance, in his discussion of the largest concordant intervals, Aristoxenus (ca. 320 BCE: Barker 1989, 139-40) said:

> … the *highest* note of the '*maiden*' [Aristoxenus's term, translator's quotation marks] aulos makes with the lowest note of the 'extra-complete' [translator's quotation marks] aulos an interval greater than the triple octave … the voice of a *child* would also be related in the same manner to that of a man … it is from *different ages* and *different dimensions* that we have discovered that the triple octave, the quadruple octave, and even greater intervals than these are concordant [my emphases].

Greater strength of adolescent and adult males would also be consistent with such a correspondence, as would greater thickness, fatness, or mass in general, and in Mesopotamian languages the opposites of both 'thin' and 'small' were employed to convey such contrasts. Moreover, such an opposition was employed in an auditory context to convey the contrast between soft and loud voices. In this way, Mesopotamians could have associated strings that produced tones of relatively high frequency with children, not only as did Aristoxenus but also the Aristotelian *Problemata* (ca. 300 BCE: Barker 1989, XI.4.24, pp. 94-95).

Accordingly, if connotations of 'thin' and 'small' in Mesopotamian languages were extended metonymically to musical tones, ***associations of lesser size, younger age, relative weakness, and softer or less prominent auditory intensity could accrue indigenously to higher fundamental frequencies in actual realizations of the Hurrian scores***. However, such associations would depend on which tones were higher and lower in actual realizations. As there is great diversity among the numbers of strings on harps and lyres in Mesopotamian pictorial representations and in the lexical record (Kilmer 1974, 72, n. 1) and as the seven string numbers are treated as a mod-7 cycle, it is far from certain which strings would produce higher and lower fundamental frequencies in actual realizations of the Hurrian scores. Nonetheless, the connotations of 'thin' and 'small' would hold for the third and fourth strings named in U.3011's account of a nine-string harp or lyre. Further, as the next sections show, aspects of U.3011's paradigmatic account in combination with the lengths of strings on Mesopotamian harps and lyres that were depicted at the time or that have survived to the present have consequences for analyses based on the twelve kinds of tuning that would satisfy U.7/80's formulation.

Higher And Lower Pitches In U.3011's Paradigmatic Account Of String-Names

In U.3011, the discursive paradigm for the nine string names is a symmetrical, retrograde, mirror ordering in which the first four strings are listed from the foremost, the last four strings are listed in reverse order from the hindmost, and the fifth string is termed, simply, 'fifth,' consistent with its being the fifth from the front and fifth from the back (irrespective of what might have been the front and back of a nine-string harp or lyre). In the following, the order in which U.3011 lists the nine strings is indicated by parentheses and the string names are unparenthesized:

(1st) fore		(9th) back
(2nd) next		(8th) 2nd of the back
(3rd) 3rd thin		(7th) 3rd of the back
(4th) 4th small		(6th) 4th of the back
	(5th) 5th	

In this paradigm, front is explicitly contrasted with back, third thin is directly contrasted with third of the back and fourth small is also directly contrasted with fourth of the back. Accordingly, the third string in the list can be construed as producing a higher fundamental frequency than the seventh string in the list, and the fourth string in the list can be construed as producing a higher fundamental frequency than the sixth string in the list.

Relative Pitches Of Strings And The Tuning Cycle

As shown above, U.7/80 prescribes for the *nitkibli* tuning the following cycle generated by intervals that span three string-steps:

4th   1st (=8th)   5th   2nd (=9th)   6th   3rd   7th

If the fundamental frequency that results from the third string is higher than the fundamental frequency that results from the seventh string and the fundamental frequency that results from the fourth string is higher than the fundamental frequency that results from the sixth string, the third string results in a tone that is higher than, and four string-steps away from, the tone in which the seventh string results, and the fourth string results in a tone that is higher than, and two string-steps away from, the tone in which the sixth string results.

On the basis of this correspondence, one might conclude precipitately that the third, fourth, sixth, and seventh strings produced fundamental frequencies that were successively lower, or as Vitale appears to have concluded, that all nine harp or lyre strings named in U.3011 were ordered from high to low from first to ninth. However, the string-names in U.3011 do not directly specify any particular registral relationships between the fundamental frequencies produced by either or both of the third and seventh strings on one hand, and on the other hand, by either or both of the fourth and sixth strings. Nor, indeed, does U.3011 provide a basis for directly comparing the fundamental frequencies produced by the first, second, fifth, eighth, or ninth strings. In fact, among the C(9,2) = 9*8/2 = 36 pairs of strings on a nine-string harp or lyre, U.3011 implies relationships for only two, namely, between the third and seventh and between the fourth and sixth .

Whereas Vitale's study does not show conclusively that these two relationships imply a single strict ordering of the fundamental frequencies produced by the first to ninth strings ordered from highest to lowest, one can more prudently set one's sights lower and specify particular consequences for a musical analysis of the Hurrian pieces that result from combining the two relationships implied by the string-names in U.3011 with the 2-Gap structure implied by U.7/80's re-tuning formulation. As Figure 15 demonstrates, merely by lowering the frequency produced by the six and seventh strings in six of the twelve possible kinds of mod-7 2-Gap tunings illustrated in Figure 6, the third and fourth

Figure 15. 12 possible *nitkibli* tunings in which the registers of the fundamental frequencies produced by strings 6 and 7 in Figure 6 are raised or lowered so that the third string is higher than the seventh (indicated by bold italics) and the fourth is higher than the sixth (indicated by underlining). Possibilities 2 and 4 are realized within a single modular interval of 1200 cents.

	d_g	**s_g**	**direction**	**strings:**						
				1	**2**	**3**	**4**	**5**	**6**	**7**
1	1	s=150	upward	0	300	***600***	-150	150	-750	***-450***
2*	1	s=150	downward	0	-300	***-600***	150	-150	-450	***-750***
3	1	L=190	upward	0	380	***760***	-190	190	-630	***-250***
4*	1	L=190	downward	0	-380	***-760***	190	-190	-570	***-950***
5	2	s=320	upward	0	640	***1280***	-320	320	-1440	***400***
6	2	s=320	downward	0	-640	***-1280***	320	-320	-960	***-1600***
7	2	L=370	upward	0	740	***1480***	-370	370	-1290	***650***
8	2	L=370	downward	0	-740	***-1480***	370	-370	-1110	***-1850***
9	3	s=500	upward	0	1000	***2000***	-500	500	-900	***1300***
10	3	s=500	downward	0	-1000	***-2000***	500	-500	-1500	***-2500***
11	3	L≈533	upward	0	1067	***2133***	-533	533	-800	***1467***
12	3	L≈533	downward	0	-1067	***-2133***	533	-533	-1600	***-2667***

strings' fundamental frequencies could be rendered higher than, respectively, the seventh and sixth strings' fundamental frequencies in each of the twelve kinds of mod-7 2-Gap tuning. In this way, any of the twelve kinds of tuning could be consistent with U.3011's names for the third and fourth strings.

On one hand, such a procedure would be consistent with U.7/80's implications that a) individual strings that are seven strings apart are part of the same degree-class insofar as they are to be altered jointly when the nine-string harp or lyre for which it prescribes re-tuning is re-tuned, and b) a string (e.g., the fourth string) that is three strings away from another string (e.g., the first string) in one direction forms the same kind of interval, i.e., *zaku* or *la zaku*, as it (e.g., the fourth string) forms with the string that is four strings away in the other direction (e.g., 8 = 1-mod-7). In other words, the degree-class ordering of strings could be considered 'helical.'

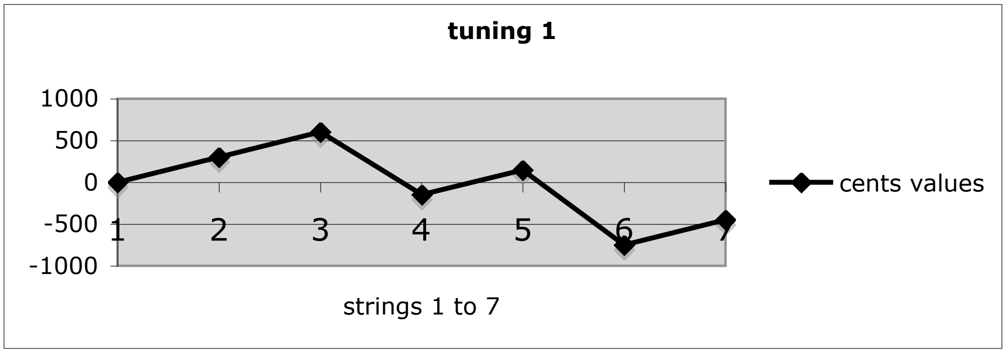
In a helical ordering, the specific magnitude of two degree-classes would correspond to the amount of rotation in one direction around the axis at the center of a cylinder on which the degree-classes would be arranged in a helix (i.e., a coil- or spring-like space). For instance, if, relative to a modular interval of 1200 cents, degree-class x was 500 cents above degree-class y, the amount of rotation from x to y would be (5/12)*360 degrees = 150 degrees clockwise or 150 degrees counterclockwise, which would be the same as 360-150 = 210 degrees in the opposite direction, i.e., respectively, counterclockwise 210 degrees or clockwise 210 degrees. Within such a helical framework, higher-than and lower-than relationships in the usual, linear sense would correspond to distances along the cylinder's axis, e.g., from left to right or bottom to top depending on the cylinder's orientation, which might be horizontal or vertical.

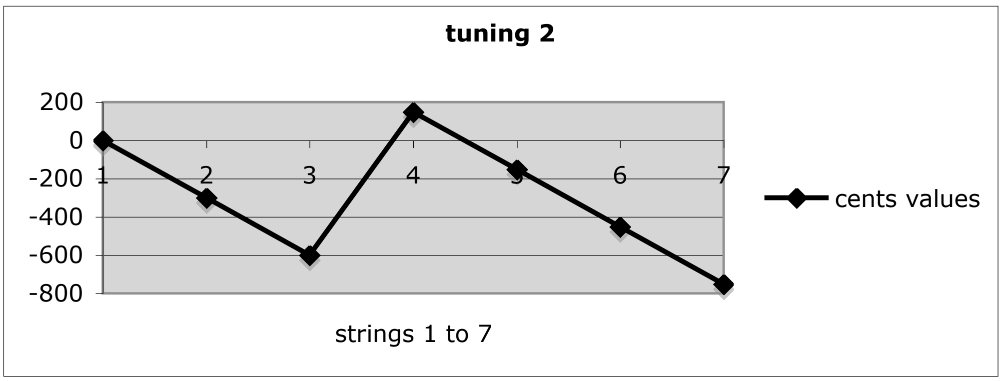
Such a model would be consistent with the values in Figure 15 but would conflict with what is known of Mesopotamian harps and lyres. In depictions of the time, the lengths of strings that are seven strings apart on a lyre differ very little, and on harps of the time the string lengths increase quite uniformly from one end of the instrument to the other. In contrast, the range of frequencies among the seven strings exceeds the modular interval's specific magnitude in all but two of the twelve tunings illustrated in Figure 15. (Tunings two and four, whose generating intervals proceed downward and span one degree, are exceptionally compact.) As well, in contrast to the similar lengths of lyre strings and the incrementally increasing lengths of harp strings, the fundamental frequencies in Figure 15 rise and fall considerably within seven-string spans. Nonetheless, as Figure 16 shows, all but two of the tunings in Figure 15 can be rendered sufficiently compact to be realized within a single modulus. However, as Figure 16 also shows, only two of the remaining ten, namely, possibilities 9 and 11 in Figure 16, can be realized within a single modulus in such a manner that their fundamental frequencies do not rise and fall considerably within a seven-string span. Indeed, as shown in Figure 17, the fundamental frequencies in possibilities 9 and 11 decrease from string to string throughout strings one to seven, and this pattern could be extended to strings 8, 9, 10, … by successively subtracting 1200 cents from the fundamental frequencies of strings 1, 2, 3, …, and above string 1, i.e., to strings 0, -1, -2, …, by successively adding 1200 cents to the fundamental frequencies of strings 7, 6, 5, … and so forth.

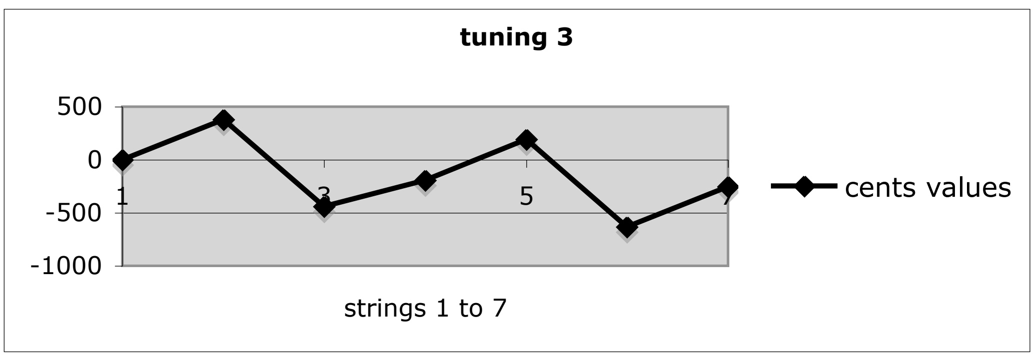
Figure 16. 12 possible *nitkibli* tunings in which the 3rd string is higher than the 7th (indicated by bold italics) and the 4th is higher than the 6th (indicated by underlining) and which are rendered as registrally compact as possible: possibilities 5 and 7 cannot be realized within a single modular interval of 1200 cents.

	d_g	s_g	direction	strings:						
				1	**2**	**3**	**4**	**5**	**6**	**7**
1	1	s=150	upward	0	300	***600***	-150	150	-750	***-450***
2	1	s=150	downward	0	-300	***-600***	150	-150	-450	***-750***
3	1	L=190	upward	0	380	***-440***	-190	190	-630	***-250***
4	1	L=190	downward	0	-380	***-760***	190	-190	-570	***-950***
5*	2	s=320	upward	0	640	***1280***	880	320	-240	***400***
6	2	s=320	downward	0	-640	***-80***	320	-320	240	***-400***
7*	2	L=370	upward	0	740	***280***	830	370	-90	***-550***
8	2	L=370	downward	0	-740	***-280***	370	-370	90	***-650***
9	3	s=500	upward	0	-200	***-400***	-500	-700	-900	***-1100***
10	3	s=500	downward	0	200	***400***	500	-500	-300	***-100***
11	3	L≈533	upward	0	-133	***-267***	-533	-667	-800	***-933***
12	3	L≈533	downward	0	133	***267***	533	-533	-400	***-267***

Figure 17. Graphic display of tunings 1 to 12 in Figure 16.







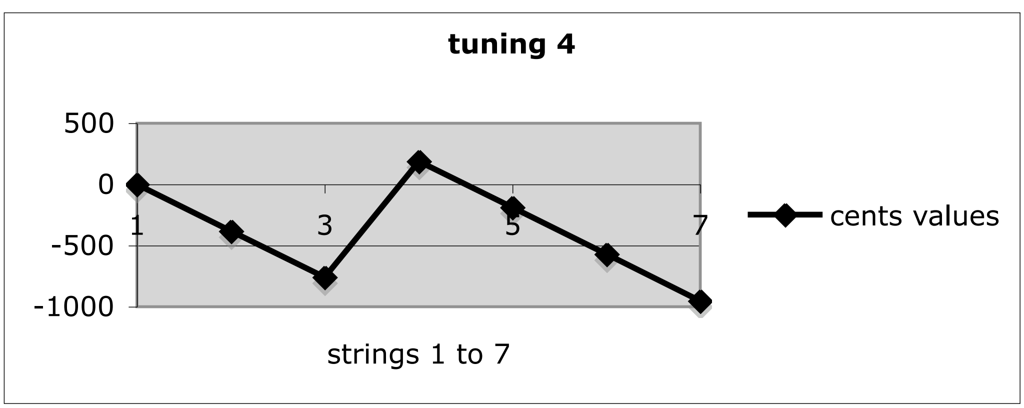
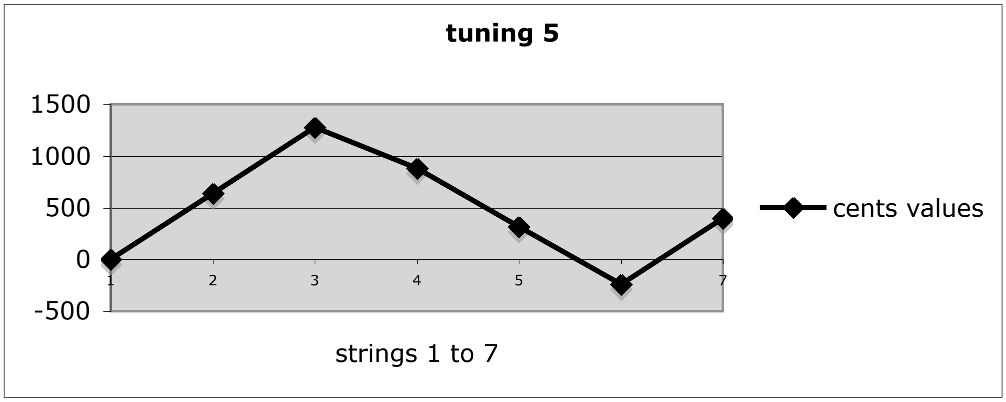
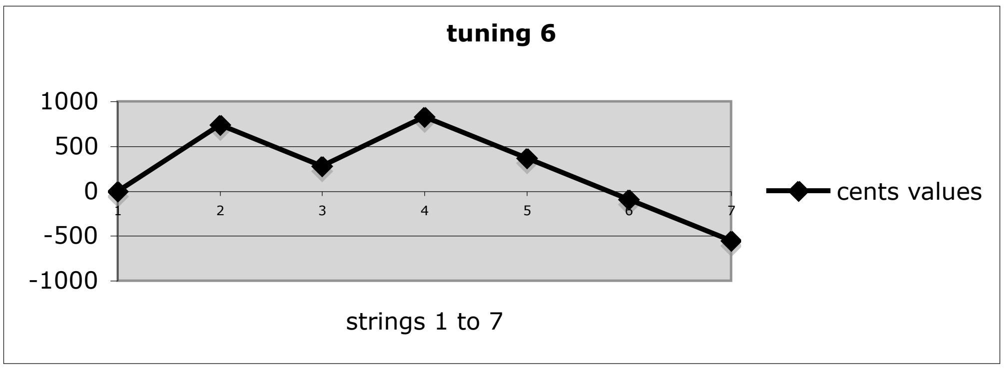
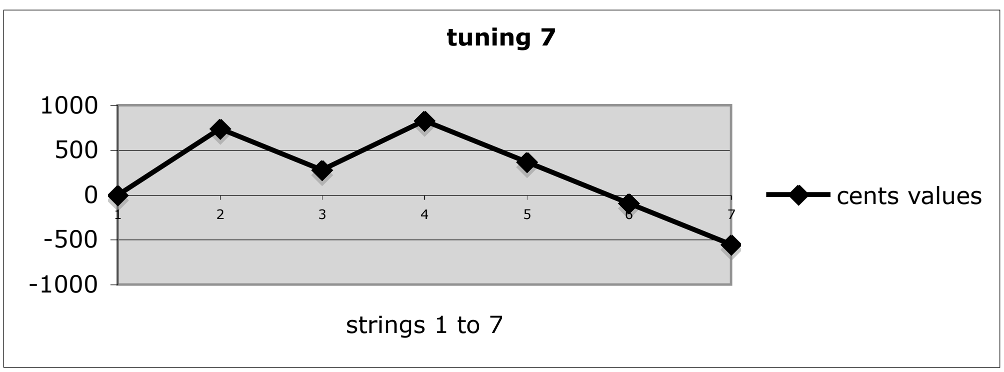


Figure 17 (cont'd).







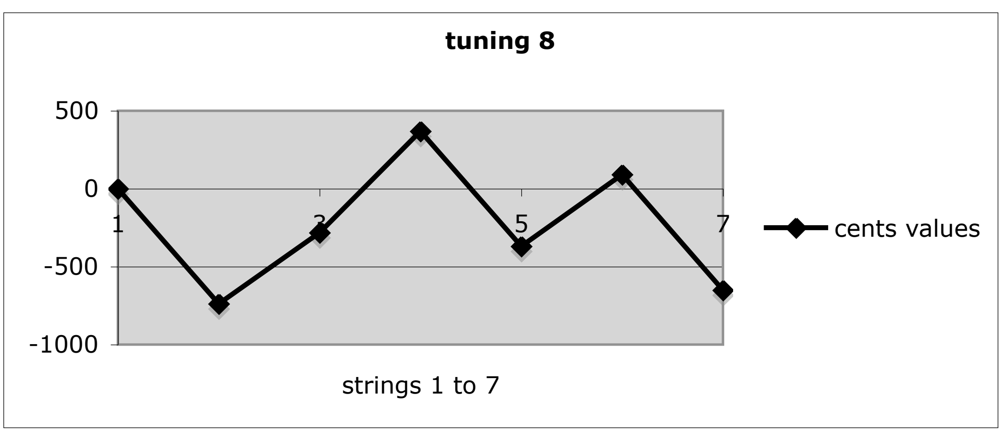
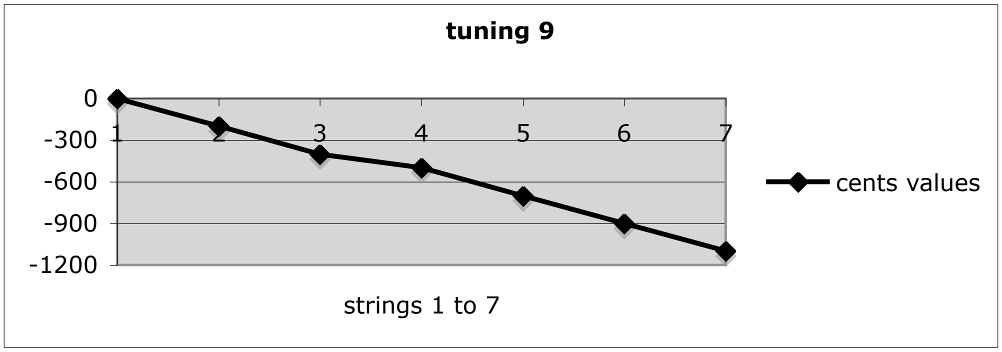
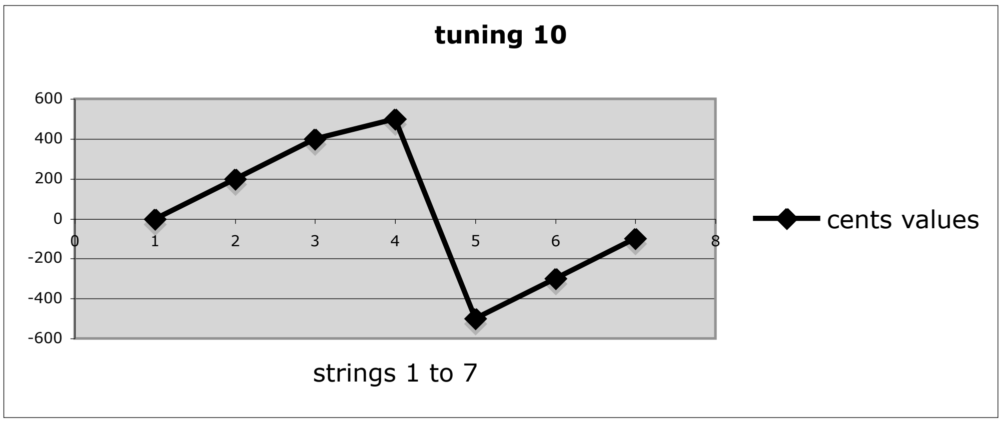
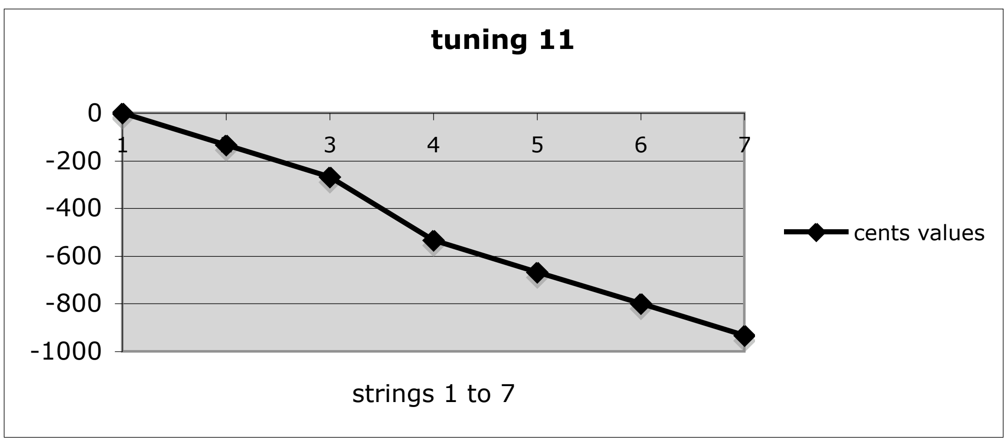
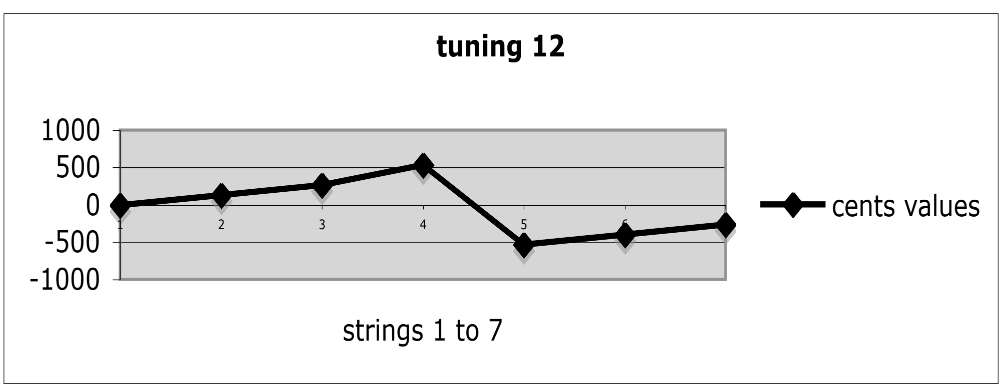


Figure 17 (cont'd).









Consequences Of Tunings 9 And 11

Consistent with these analogical relationships between fundamental frequencies and string numbers would be the visual-spatial arrangement of harp strings from shorter to longer, an ordering that would be visible to performers and listeners, and the resulting analogical relationships between fundamental frequencies and the visual-spatial arrangement of harp strings would hold for instruments and ensembles comprising many more than nine strings. In this regard, the referents of 'front' and 'back' in U.3011 would be clarified, 'front' corresponding to high-frequency strings and 'back' to low-frequency strings. As well, depictions of Mesopotamian harps would lead one to identify high-front strings with those closest to a person plucking them and low-back strings with those most remote from the performer. Accordingly, a further consequence of tunings 9 and 11 would be a correspondence between, on one hand, tactile-motor acts ranging from closer to more remote, relative to the performer, and on the other hand, the ordering of string lengths from smaller to larger and the corresponding ordering of their fundamental frequencies from high to low.

As in the single tuning advanced by Vitale and others (West 1994, 177-79; Gurney 1994, 102-06; Gurney and West 1998; Dumbrill 2005; Hagel 2005, xx; cf., however, Crocker 1997; Kilmer 2009-10, Table 2), the ordering of the strings' fundamental frequencies in tunings 9 and 11 would be downward from the first to the ninth string (and beyond in both directions). Moreover, in tuning 9, the generating interval's specific magnitude would be between (2/5)*sm and (3/7)*sm, as in all the tunings that Assyriologists have proposed. However, their proposals have been much more precise than what is considered here.

Previous studies have assumed that the specific magnitudes of the modular interval and the generating interval corresponded to the fundamental-frequency ratios 2/1 = 1200 cents and 4/3 ≈ 498 cents, respectively. In contrast, the 1200-cents value for the modular interval is treated here as merely illustrative, and even within illustrations that employ the 1200-cent value, the most precise claims concerning the twelve kinds of mod-7 2-Gap tunings discussed above involve such ranges of values as (2/5)*sm = 480 cents to (3/7)*sm ≈ 514 cents and (3/7)*sm ≈ 514 cents to sm/2 = 600 cents.

Also possible only in the two upward-3-mod-7 tunings is a further consequence: specifically, an additional parallel between Mesopotamian music and 2-Gap cycles in other cultures. In particular, the number of steps in the generating interval, d_g, of each of the 2-Gap cycles outlined in Figure 9 (above) is the same as in possibilities 9 and 11 in Figure 16, namely, $(d_m \pm 1)/2$. In the 7-step 2-Gap cycles, $d_m = 7$ and the number of steps in the generating interval is $(7 \pm 1)/2 = 4$ or 3; in the 5-step 2-Gap cycles, $d_m = 5$ and the number of steps in the generating interval is $(5 \pm 1)/2 = 3$ or 2. Insofar as the number of steps in the generating interval is as close an approximation to 1/2 as is possible among natural, counting numbers (cf. Rahn 1977, 45), the number of steps in the modular interval, $(d_m \pm 1)/2$ can be considered a 'half-modular' interval and each of these cycles can be considered not only a 2-Gap cycle but more precisely a half-modular 2-Gap cycle.

To be sure, the possible values for the half-modulus's specific magnitude might seem to span ranges that are too large to justify the notion of such cycles being 'more precisely' half-modular. However, the ranges of, e.g., 480 to ~514 cents and ~514 to 600 cents jointly exclude 480/600 = 80% of the possible specific magnitudes for a 7-step 2-Gap cycle's generating interval, and even if one takes into account the exclusion of the ranges from 200 to 300 cents and from 400 to 480 cents (as mentioned above in the discussion of Figure 6), possible half-modular generating intervals jointly span only (600-480)/(600-((300-200)+(480-400)) = 120/420 ≈ 28.6% of the remaining range.

Whereas the relationships of sameness and analogy among string-numbers that are discussed in the first part of this study would hold among strings ordered from high to low in any of the 12 tunings illustrated in Figure 16, relationships of adjacency discussed in the first part would hold only among strings ordered from high to low in tunings 9 and 11. In this way, tunings 9 and 11 could result in numerical relationships being re-interpreted as perceived relationships: specifically, auditory relationships. As the next sections show, such perceived relationships can be construed in terms of Gestalt Grouping Principles. The following sections discuss the Gestalt Grouping Principles of Similarity, Proximity, and Common Fate and consider their perceptual application to an analysis of h.6 framed in terms of tunings 9 and 11.

FROM SAMENESS, ADJACENCY, AND ANALOGY TO SIMILARITY, PROXIMITY, AND COMMON FATE

Sameness, adjacency, and analogy relationships in the Hurrian pieces as a group and in h.6 specifically can be construed in terms of phenomena or acts of perception that are 'bottom-up,' and hence, arguably not culturally specific (Jan 2004, 71). Since the 1920s, such perceptual acts have been formulated in terms of the Gestalt Grouping Principles of Similarity, Proximity, and Common Fate.

According to the Principle of Similarity, things that are perceived as the same also tend to be perceived as parts of a group. In an adaptation of one of Max Wertheimer's (1923, 312) classic illustrations, the 6 circular things of Figure 18(a) are seen not only as

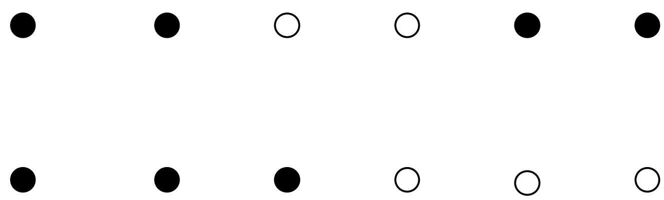
Figure 18. Gestalt Grouping "Principle of Similarity" (*Faktor der Gleichheit*) illustrated by means of visual displays (after Wertheimer 1923, 312):

a) the six circular things are perceived as constituting three groups by virtue of, from left to right, the first and second and the fifth and sixth things being perceived as filled and the third and fourth things being perceived as unfilled, and contrariwise, by virtue of the first and second and the fifth and sixth things being perceived as differing in 'filledness' from the third and fourth things;

b) the six circular things are perceived as constituting two groups by virtue of, from left to right, the first, second, and third things being perceived as filled and the fourth, fifth, and sixth things being perceived as unfilled, and contrariwise, by virtue of the first, second, and third things being perceived as differing in 'filledness' from the fourth, fifth, and sixth things.

a)

b)



6 separate circular things but also as 3 pairs of circular things: in contrast, the 6 circular things of Figure 18(b) are seen not only as 6 separate circular things but also as 2 groups of 3 circular things. According to the Similarity Principle, the perceived groupings in Figure 18(a) are a result of the sameness of the 1^{st} and 2^{nd}, 3^{rd} and 4^{th}, and 5^{th} and 6^{th} circular things with regard to filledness, and in Fig. 18(b), the sameness of the 1^{st}, 2^{nd}, and 3^{rd} circular things and the sameness of the 4^{th}, 5^{th}, and 6^{th} circular things, again with regard to filledness. Although Wertheimer does not specify such details, all 6 circular things in both visual displays would be perceived as a group as a result of their sameness with regard to circularity, size, and location on the vertical axis.

According to the Principle of Proximity, things that are perceived as relatively close to each other tend to be perceived as parts of a group. In an adaptation of another of Wertheimer's (1923, 308) classic illustrations, the 6 circular things of Figure 19(a) are seen not only as 6 separate circular things but also as 3 pairs of circular things: in contrast, the 6 circular things of Figure 19(b) are seen not only as 6 separate circular

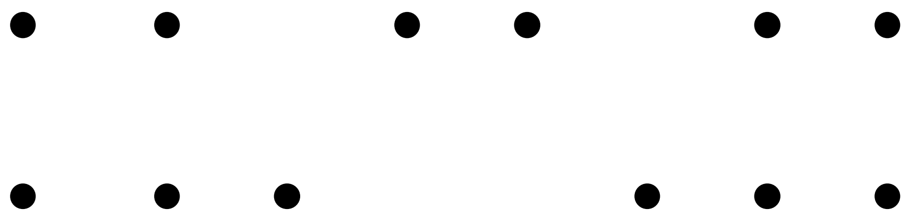
Figure 19. Gestalt Grouping Principle of "Proximity" (*Faktor der Nähe*) illustrated by means of visual displays (after Wertheimer 1923, 308):

a) the six circular things are perceived as constituting three groups by virtue of, from left to right, the first and second, third and fourth, and the fifth and sixth things being perceived as relatively close together, and contrariwise, by virtue of the second and third and the fourth and fifth things being perceived as relatively far apart;

b) the six circular things are perceived as constituting two groups by virtue of, from left to right, the first, second, and third, and the fourth, fifth, and sixth things being perceived as relatively close together, and contrariwise, by virtue of the third and fourth things being perceived as relatively far apart.

a)

b)



things but also as 2 groups of 3 circular things. According to the Proximity Principle, these perceived groupings are a result of the greater distance between the 2nd and 3rd circular things and between the 4th and 5th circular things in Figure 19(a), and in Figure 19(b) the greater distance between the 3rd and 4th circular things. Again, although Wertheimer does not specify such details, all 6 things in both visual displays would be perceived as a group as a result of their sameness with regard to circularity, size, location on the vertical axis, and filledness.

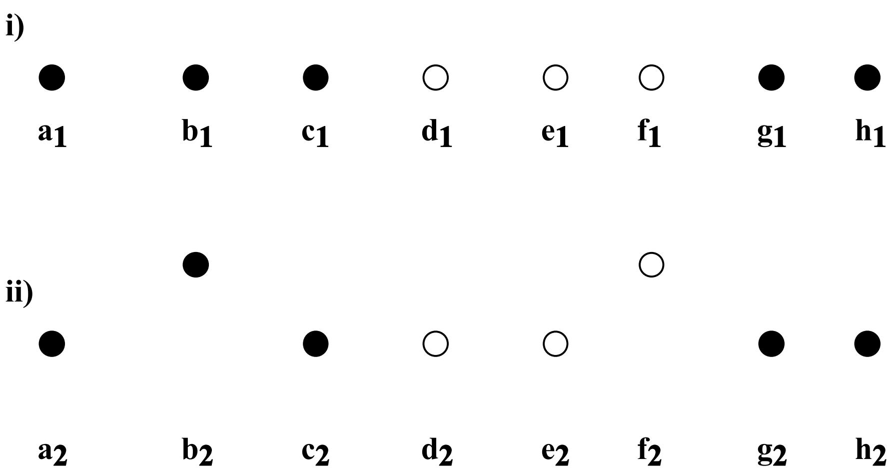
In an adaptation of yet another of Wertheimer's classic illustrations (1923, 315), the 2nd and 6th circular things from the left in Figure 20(a) (labeled b_1 and f_1) would be perceived as part of a group comprising all 8 circular things, for all 8 things in Figure 20(a) are the same with regard to circularity, size, and location on the vertical axis; as well, b_1 and f_1 would be perceived as parts of 2 of the 3 groups by virtue of Proximity as well as Similarity with regard to filledness (labeled a_1, b_1, and c_1; d_1, e_1, and f_1; and g_1

Figure 20. Gestalt Grouping Principle of 'Common Fate' (*Faktor des gemeinsamen Schicksals,*cf. also *Faktor der guten Kurve* and *durchgehende Gerade*) illustrated by means of visual displays (after Wertheimer 1923, 315):

a) the 8 circular things are perceived as constituting a 3 groups by virtue of Similarity;

b) the 8 circular things are perceived as constituting 3 groups by virtue of Similarity and 2 groups relative to the x-axis.

If Figures 20(a) and 20(b) alternate as in the animation of Figure 20(c), b_1 and b_2 and f_1 and f_2 are perceived as a single thing, i.e., a single pair of circular things, that moves relative to the group that comprises the other 6 things.



and h_1). The 2nd and 6th circular things from the left of Figure 20(b) (labeled b_2 and f_2) would be perceived not only as parts of the same groups as b_1 and f_1 by virtue of Proximity and Similarity but also as parts of a group in their own right by virtue of Proximity and Similarity with regard to their location on the vertical axis.

Although the difference in grouping between Figure 20(a) and 20(b) can be accounted for by Proximity and Similarity if the visual displays are considered severally, Wertheimer indicates that if Figure 20(a) alternates with Figure 20(b) (as in the animation of Figure 20(c)), the 2nd and 6th circular things from the left are perceived as a single group, i.e., b_3+f_3, where b_3=b_1+b_2 and f_3=f_1+f_2, that 'moves' vertically in contrast to the

1^{st}, 3^{rd}, 4^{th}, 5^{th}, 7^{th}, and 8^{th} things, which remain where they are, i.e., as the group $a_1+c_1+d_1+e_1+g_1+h_1$.

According to Wertheimer's Principle of Common Fate, perceiving the 2^{nd} and 6^{th} circular things in the two parts of Figure 20 as a single group is a product of their apparently joint 'motion.' The 2^{nd} and 6^{th} circular things do not 'really' move between the displays of Figure 20(a) and Figure 20(b); instead, they are merely perceived as moving.

Similarity, Proximity, And Common Fate In H.6

In a general formulation of the Gestalt Grouping Principles, Similarity, Proximity, and Common Fate correspond to the relationships of, respectively, sameness, adjacency, and analogy discussed in the first part of this study. Among degree-classes, sameness is the limiting case of maximal Proximity, less proximate degree-classes differing by 1, 2, or 3 steps. Intervals that are both generically and specifically the same are similar to a greater extent than intervals that are merely generically the same.

With regard to time, simultaneity (e.g., within a single instance of the interval realized by a particular string-pair) is the limiting case of maximal Proximity. Intervals that, among all intervals, are immediately successive, including intervals of the same specific magnitude, are temporally more proximate than intervals that are immediately successive merely among intervals of the same specific magnitude.

Analogy corresponds to Common Fate. E.g., in Figure 20, $b_1:b_2::f_1:f_2$ and $b_1:f_1::b_2:f_2$ with regard to location. Accordingly, although nothing actually 'moves' in Figure 20, b+f is perceived as moving. Moreover, $a_1:a_2::c_1:c_2$ and $a_1:c_1::a_2:c_2$, $a_1:a_2::d_1:d_2$ and $a_1:d_1::a_2:d_2$, and so forth, so that a+c+d+e is perceived as remaining where it is, just as sameness is the limiting case of maximal analogy. Arguably, though degree-classes and intervals do not actually 'move' in the Hurrian pieces, or in music generally, certain degree-classes and intervals can be perceived as moving or remaining where they are as a consequence of analogical relationships among their instances.

According to the Similarity Principle, the intervals in the passage 25…25 of h.6 would be heard as a single thing, a pair of intervals, a group, rather than merely as 2 instances of an interval. Within the passage 25…25, the intervals are the same with regard to their generic magnitudes, their generic-specific magnitudes, and the particular degree-classes they comprise. Moreover, 25…25 would be heard as a group that remains where it is, rather than merely as two intervals at two different times. So too would 14…14. To a smaller extent, the intervals in the 25…62 passage would be heard as a single pair of intervals that are the same with regard to their generic and generic-specific magnitudes and with regard to degree-class 2, which is common to both 25 and 62 (Figure 21).

The 4^{th}s in these passages are, among all the 4^{th}s in h.6 but not among all the intervals in h.6, immediately successive. Considered severally, the 36s at the end of h.6 are immediately successive neither among all the 4^{th}s nor among all the intervals; similarly, the 25s at the end of h.6 are immediately successive neither among all the 4^{th}s nor among all the intervals. Nonetheless, the 36s at the end of h.6 are the same with regard to their generic magnitude, their generic-specific magnitude, and both of their degree-classes, as are the final 25s of h.6. In contrast, the 4^{th}s in the succession 62…73…14 are immediately successive with regard to all the 4^{th}s of h.6 and are the same with regard to both their generic magnitude and their generic-specific magnitude, but not with regard to the degree-classes they comprise (Figure 22).

Figure 21. Grouping of 25…25, 14…14, and 25…62 by degree-class Similarity.

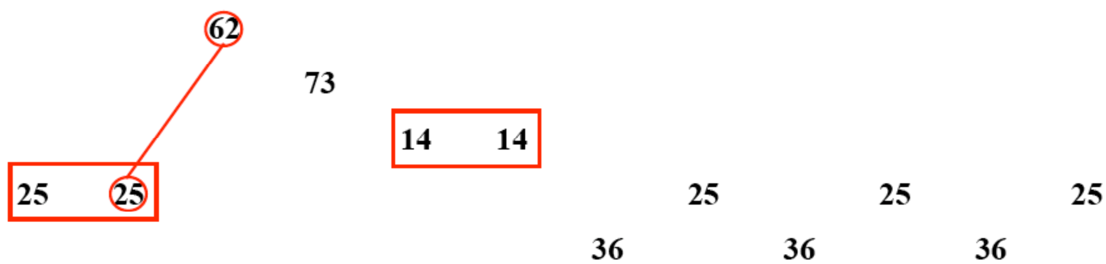
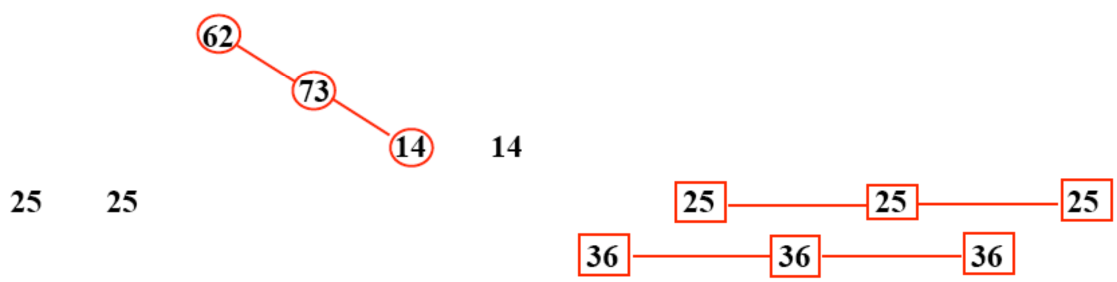


Figure 22. Grouping of 25…25…25 and 36…36…36 at the end of h. 6 by Similarity and of 62…72…14 in the middle of h.6 by Common Fate and, as a consequence of tuning 9 or 11, by Proximity.



Whereas immediate succession with regard to all the 4th s of h.6 is an instance of temporal adjacency, or, in terms of Gestalt Grouping, the Proximity Principle, immediate succession with regard to all the intervals of h.6 is an instance of temporal adjacency or Proximity to a greater extent. Further, whereas the respective degree-classes within the 4th s in the passage 25…25 are maximally proximate, albeit within a cyclic framework of mod-7 degree-classes, as are the 4th s in the passage 14…14, the degree-classes within the 4th s in the passage 62…73…14 are proximate to a lesser extent.

Although the 2-Gap tuning described by U.7/80 specifies 2 specific-magnitude categories for each generic interval, in the absence of an assumption, conjecture, or conclusion concerning the precise specific-magnitude of the generating interval one cannot determine which 2^{nd}s are larger or smaller than the other 2^{nd}s. Accordingly, one cannot compare the Proximity of 62 and 73 with the Proximity of 73 and 14. Nonetheless, one can specify that 62 and 73 are as proximate as 36 and 25, and all these pairs of immediately successive 4^{th}s, namely, 62 and 73, 73 and 14, and 36 and 25, are less proximate than the 4^{th}s in the passages 25…25 and 14…14.

By virtue of being only 1 step apart in tunings 9 and 11, the immediately successive 4^{th}s at the end of h.6 would be parts of a group with regard to Proximity, insofar as no interval is immediately repeated and these 4^{th}s would proceed by the smallest possible amount relative to the tunings' degree-classes, namely, by a single degree-class (Figure 23). By the same token, immediately successive 3^{rd}s throughout h.6 would be parts of various groups. Such groups would include the 3^{rd}s that alternate between adjacent degree-classes (Figure 24) and the 3^{rd}s whose degree-class numbers increase transitively by a single degree-class (Figure 25).

In tunings 9 and 11, the intervals that comprise degree-classes 13, 46, and 57 are the same as each other in specific magnitude and differ from the intervals that comprise degree-classes 24, 35, 61, and 72, which are the same as each other in specific magnitude. On one hand, this difference would result in h.6's immediate stepwise successions of 3^{rd}s being variegated or striated in contrast to the piece's non-immediate stepwise successions of 4^{th}s. On the other hand, the generic-specific sameness of 61 and 72 at the end of the

Figure 23. Grouping of 36-25-36-25-36-25 at the end of h.6 by Common Fate and Similarity, and as a consequence of tuning 9 or 11, by Proximity.

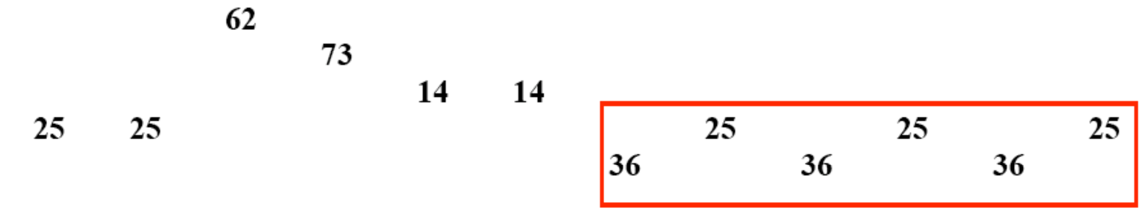


Figure 24. Grouping of 57s and 61s in the middle of h.6 by Similarity and Common Fate, and as a consequence of tuning 9 or 11, by Proximity.

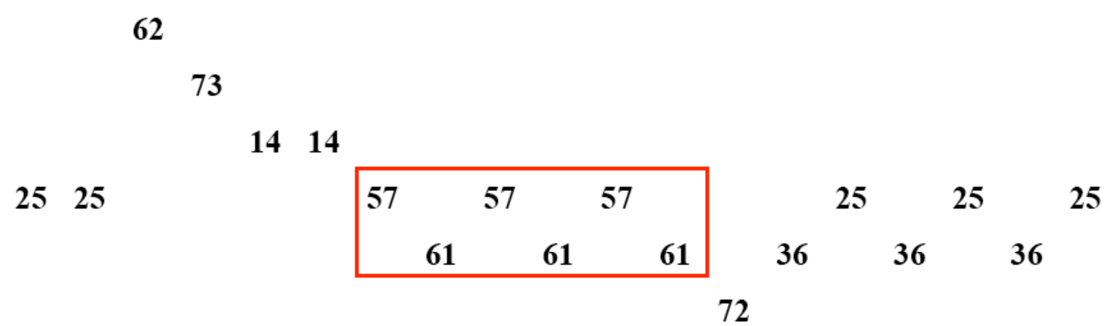
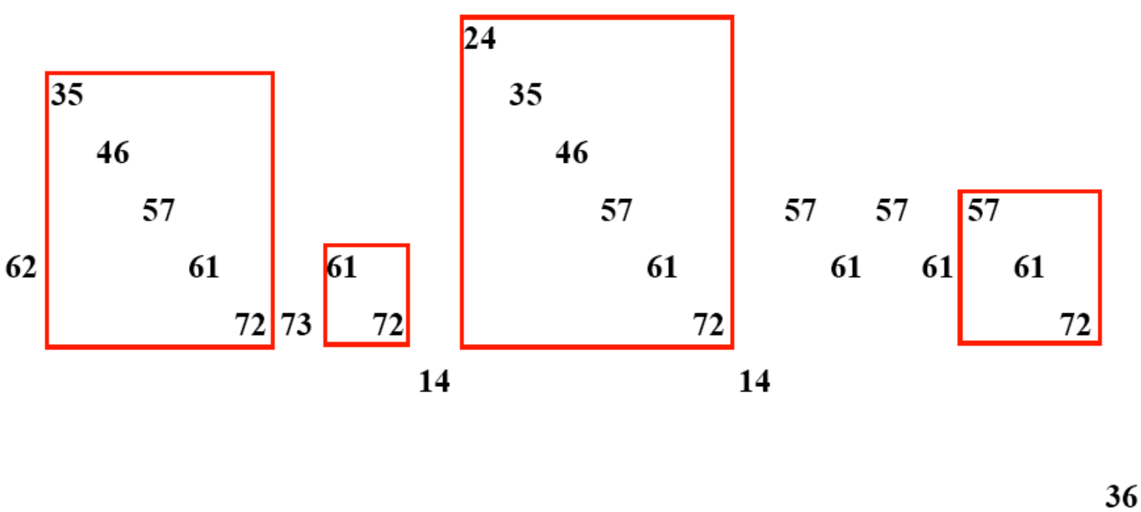


Figure 25. Grouping of intervals by Similarity and Common Fate, and as a consequence of tuning 9 or 11, by Proximity.



last 4 3rd successions stands out in contrast to their diverse beginnings: yet another instance of ***temporal asymmetry*** (Figure 26).

As depicted in Figures 21 to 26, the groups just considered are 'static.' Construed in terms of Common Fate, the relationships on which such groups are based can be considered 'dynamic' insofar as degrees and intervals, individually and in groups, 'move' throughout the piece (Figures 27 and 28).

CONCLUSION

Deciphering h.6 and the other 34 Hurrian scores depends on having translated the 3 main cuneiform sources for our knowledge of the signs employed in their notation. Translating these sources depends in turn on their paradigmatic structure and the numerical ordering of their names for strings and string-pairs. In this regard, the

Figure 26. Grouping of generic intervals in immediate succession by temporal Proximity as well as generic Similarity and Common Fate with regard to pitch, and as a consequence of tuning 9 or 11, by generic Proximity. Contrasting specific magnitudes of intervals that would be thirds in tuning 9 or 11 are indicated by italics *vs.* underlining; generic-specific Similarity of intervals that would be fourths in tuning 9 or 11 is indicated by underlining and bold typeface.

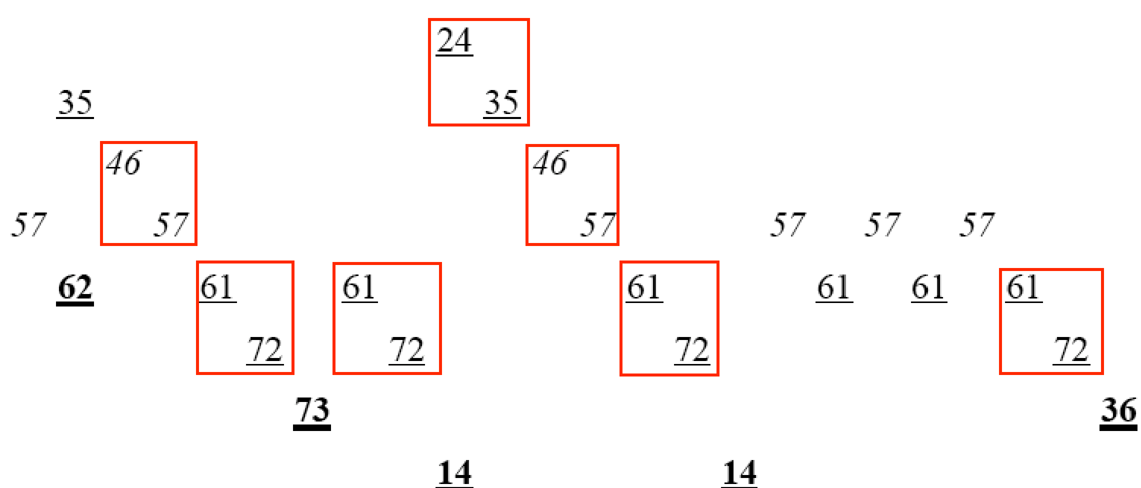


Figure 27. Proximity and Common Fate groupings among immediately successive degree-classes (single arrows) and intervals that would be thirds (double arrows) in tuning 9 or 11 in h.6.

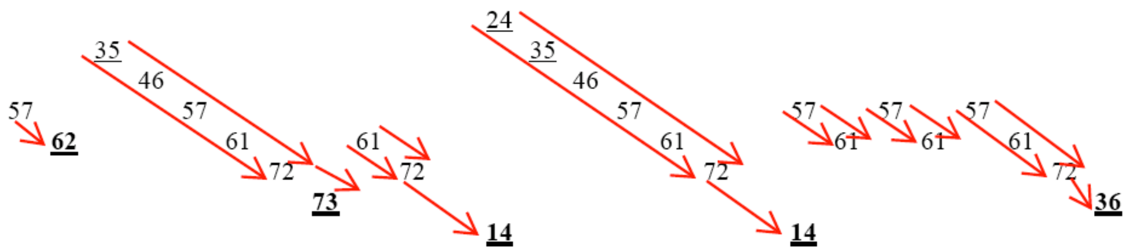
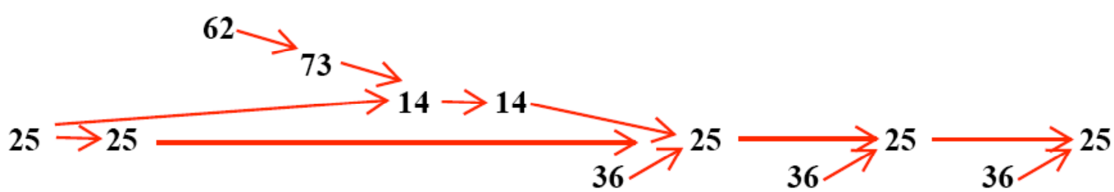


Figure 28. Common Fate groupings among intervals of h.6 that would be small fourths in tuning 9 or large fourths in tuning 11.



Mesopotamian tradition is the first recorded instance of ordered and paradigmatic discourse about music. Moreover, with regard to analysis, relationships of adjacency, which can be interpreted in terms of the Gestalt Grouping Principle of Proximity, are relationships of ordering; as well, relationships of analogy, which can be interpreted in terms of the Gestalt Grouping Principle of Common Fate, are paradigmatic, and relationships of sameness, which can be interpreted in terms of the Gestalt Grouping Principle of Similarity, can be considered special, extreme cases of Common Fate and Proximity.

Usually, musical analysis is based on much more detail than the Hurrian pieces provide. E.g., ethnomusicological transcription of recorded sounds results in quite precise notations of pitches and durations, and in historical musicology musical analysis takes as its point of departure pitches and durations that have already been notated, or if absolute pitch and tempo have not been documented, the point of departure comprises pitch- and time-intervals. In contrast, the 3 main sources for our knowledge of Mesopotamian notation result in 12 possible 2-Gap configurations of degree-classes for h.6 and the other 2 pieces that specify *nitkibli* as their tuning, rather than pitches or pitch-intervals, and temporal relationships that specify only the temporal ordering of pairs of degree-classes rather than the time-intervals between successive degree-classes or degree-class-pairs.

Despite such a limited starting-point for analysis, the Hurrian scores convey a considerable amount of structure. The quantitative implication of the word ‘amount’ is borne out by the numbers of relationships of sameness, adjacency, and analogy in the pieces that can be counted quite precisely in an analysis. Insofar as such relationships maximize analytical ‘wholeness’ and ‘richness’ (Rahn 1983, 51) and to the extent that the basis of these relationships is parsimonious, such an approach to analysis realizes the ontological value of ‘economy,’ which Nelson Goodman (1977, 49) proposed as a systematic desideratum that corresponds to ‘fuel consumption,’ i.e., ‘miles per gallon.’

The 7-step 2-Gap structure of Mesopotamian tuning has had parallels in subsequent traditions as widely separated in time and space as ancient Greece, Europe from the Middle Ages onward and Indonesia. As well, 2-Gap structures have been employed in the 5-step cycles of anhemitonic pentatonic, 5-step *pélog*, and 5-step equiheptatonic. Further, 2 of the 12 possible 7-step 2-Gap structures have half-modulus generating intervals, an aspect of structure shared by all of the 2-Gap tunings just mentioned. In any event, Mesopotamian music in general and the Hurrian pieces in particular are the first recorded instances of such structure.

Both of the half-modular 2-Gap possibilities would result in analogical relationships among string-lengths on a Mesopotamian harp and tactile-spatial aspects of performance including the distances a performer would reach to pluck individual strings. Such analogical relationships would also be extended to comprise higher and lower degrees, which, in turn, would connote gendered contrasts between childhood and adulthood, size etc. in Akkadian. Further, whereas all 12 of the possible 7-step 2-Gap tunings are interpretable in terms of the Gestalt Grouping Principles of Similarity and Common Fate that identify particular groups of tones within h.6, including various 'motions' of such groups, the 2 half-modular tunings would amplify such groups with regard to the Proximity Principle.

Without adopting such anachronistic, and arguably ethnocentric, concepts as the parallel 5ths, triads, just intonation, leading tones, and the tonic invoked by Hagel (2005, 312-15, 321-22), one can make sense of h.6 and specify ways in which its features are shared by the remaining 34 Hurrian pieces. In this regard, the statistical homogeneity of the *nitkibli* and non-*nitkibli* pieces indicate that irrespective of which of the 12 tunings might have been employed in the non-*nitkibli* pieces, the entire repertoire constituted a single idiom.

Features of Mesopotamian tuning that remain to be considered include possible specific magnitudes of the modular and generating intervals and the possibility that lines 1 to 11 and 13 to 19 of U.7/80 specified, respectively, raising and lowering particular pitches. Also to be considered are consequences of the possibility that the Hurrian pieces were created on, or subsequently realized by, a 9-string instrument of the sort described in U.7/80 and U.3011. As Hagel (2005, 311-17, 320) has suggested, certain tendencies among the 2- and 3-mod-7 string-pairs 3rds might have been an artifact of such a possibility. In any event, such possibilities are to be assessed in the first instance in terms of their analytical consequences.

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