The Hurrian Pieces, ca. 1350 BCE: Part One—Notation and Analysis*

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The earliest known pieces of music were notated on cuneiform tablets ca. 1350 BCE. Excavated at the site of the ancient city of Ugarit near present-day Ras Shamra in northwestern Syria, these tablets were transcribed and transliterated more than 40 years ago by Emmanuel Laroche (1955, 1968). Thereupon, David Wulstan (1968) and Oliver R. Gurney (1968) advanced a basis for interpreting the tablets musically. Since then, scholars of Mesopotamian and Ancient Greek music have proposed several modern staff-notation editions of the single cuneiform score that has survived intact (e.g., Wulstan 1971; Kilmer 1974; Duchesne-Guillemin 1975, 1980; Vitale 1982; West 1994; Dumbrill 2005). Moreover, scholars have reported various empirical tendencies among several of the 35 cuneiform scores that have survived (e.g., West 1994, Dumbrill 2005, Hagel 2005; Halperin 2010).

Such analyses have assumed that the tuning system relevant to the Hurrian scores was based on an octave and a fifth whose fundamental-frequency ratios were, respectively, 2:1 and 3:2. However, no Mesopotamian source specifies such ratios. Accordingly, rather than assume such values, the present study analyzes the scores in terms of aspects of Mesopotamian music that are much less conjectural: in particular, the

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names and numerals that were employed to identify strings on a harp or lyre and the very general directions that were provided for tuning such strings.

In addition to the absence of clear information concerning the fundamental-frequency ratios employed in realizing the Hurrian scores are other impediments to musical analysis. There remains considerable uncertainty concerning the Hurrian-language words that precede each of the scores as well as several cuneiform characters that are interspersed among the main, string-based symbols of the scores' original cuneiform notation. For these, recent studies by Richard Dumbrill (2005) and Stefan Hagel (2005) have supplied conjectural accounts, which, however, presume that the tuning by which the Hurrian scores were realized was based on the fundamental-frequency ratios 2:1 and 3:2. Nonetheless, the string-based symbols and their temporal ordering are sufficiently clear to serve as a basis for characterizing the single score that is intact, namely, the score identified by Assyriologists as 'h.6.' Further, the string-based symbols and their temporal ordering are also an adequate basis for comparing h.6 with the other 34 scores that have been identified so far, even though these 34 scores are highly fragmentary and notationally discontinuous, due to considerable damage to the original cuneiform tablets and both partial and complete destruction of many of their original cuneiform characters during the past three millennia (Figure 1).[1]

[1] The usual format for the cuneiform tablets on which the Hurrian scores were inscribed was oblong, in contrast to the somewhat triangular shape of what remains of h.7. Line 11 is one of 3 surviving Akkadian colophons that specify a *nitkibli* tuning for the score. The first five lines are surviving portions of a Hurrian text that might have been recited or sung in connection with the musical notation that follows on lines 7 to 10. Notwithstanding elaborate and uncorroborated conjectures by such writers as Dumbrill (2005, 115-32), no clear relationship has been established between the syllables of the Hurrian texts and the string-pairs that follow them.

Akkadian and Hurrian (or Hurrianized Akkadian) terms for string-pairs (i.e., *kablite*, *irbute*, *shahri*, *shashate*, *nitkibli*, *titarkabli*,… and *kitme*) appear in lines 7-10. What the individual numerals (i.e., 3, 1, 1, 2?, 4, 1, 2, 3, 1) in these lines refer to is far from certain. The main conjectures have been that a) all the

Figure 1. Discontinuous state in which 34 of the 35 extant Hurrian scores survive, illustrated by Laroche's (1968, 465, 488) transcription (i.e., diplomatic edition) and transliteration of the extant verbal text and musical notation of h.7.

string-pair numerals	transliteration of original tablet
	]-ni wa-x[
	]x -at z[i?-
	-]nu-ḫa-at [
	-]l-la-at ḫ[a?-
	x-we-ni-wa-al a?[-
	az-za-mi-ra be-ni-[
25 72	kab-li-te 3 ir-bu-te 1 [
57 61 57	sh-aḫ-ri 1 sha-ash-sha-t[e] 2 ? sha-[
14 24 35	ni-[i]t-kib-li 4 ti-tar-kab-li 1 ti-t[i?
61 72 36	sha-ash-sha-te 2 ir-bu-te 3 ki-it-me 1 [
	[a]n-nu-ú za-am-ma-ash-sha ni-it-kib-li za-lu-zi [

transcription of original tablet

NAMES AND NUMERALS FOR PAIRS OF STRINGS AND INDIVIDUAL STRINGS ON A HARP OR LYRE

The most certain components of the 35 surviving scores are names for pairs of strings on a harp or lyre. These string-pair names are employed as the main means of notation in the original scores and are identified in another Mesopotamian tablet with pairs of numerals. Each pair of numerals and each string-pair name corresponds to one of 14 pairs of strings that are named in the cuneiform tablet CBS 10996, column i (ca.1500 BCE or ca. 500-0 BCE: Kilmer 1960, plate 83; Duchesne-Guillemin 1963, 3-7; Kilmer 1965, 265-67; Duchesne-Guillemin 1965; Wulstan 1968, 215-16; Kilmer 1971, 132-33, where it is specified as having originated in the Kassite period, mid-second millennium, or in the

numerals except 10 designate the number of pitches between the highest and lowest tones that result from realizing the preceding string-pair that are to be realized melodically (e.g., Wulstan 1971, 377-80), and b) the numerals specify the number of times the preceding string-pair is to be realized as a simultaneity (e.g., Kilmer 1974). Unlike the string-pair terms and the indications of *nitkibli* tuning, the referents of the numerals are uncorroborated by other sources concerning Mesopotamian music. Nonetheless, one can observe that single numerals appear after single string-pair terms, suggesting that each numeral somehow refers to the string-pair term it follows.

Neo-Babylonian period, mid- to late-first millennium; Kilmer 1984, 69; West 1994, 162-63).

CBS 10996 identifies each string-pair it names with the Mesopotamian names for two of the first seven strings of a nine-string harp or lyre (Sumerian, ^{gis}ZÀ.MÍ; Akkadian, *sammû*, in U.7/80: Gurney 1968, 229—the transliterations in the present study are as close as standard fonts allow to the transliterations that appear in the publication that is first cited for a particular quotation). The first seven strings of such a nine-string harp or lyre, as well as the eighth and ninth strings, are named in another cuneiform tablet: U.3011, column i (Wulstan 1968, 215-17; Kilmer 1971, 133-34; Gurney 1974, 126; Shaffer 1981, 79-81, where the parallel text of cuneiform tablet N 4782 is dated ca. 1750 BCE; Finkel and Civil 1982, 249-54, where U.3011 is dated ca. 1500 BCE). As well, CBS 10996 identifies pairs of the first seven string-names with pairs of numerals from 1 to 7. For the convenience of non-Assyriologists, the present study generally refers to string-pairs by means of pairs of Hindu-Arabic numerals (e.g., '6-and-2,' or later in this study, '62'), rather than transliterations of their Akkadian names (e.g., *ishartum*: Kilmer 1971, 133).

Much of the analysis of h.6 and much of the comparison of h.6 with the other 34 Hurrian scores that follows is framed in terms of the 14 string-pairs that are specified and named in CBS 10996. Seven of these string-pairs comprise two strings whose numerals differ by 2-modulo-7: 1-and-3, 2-and-4, 3-and-5, 4-and-6, 5-and-7, 6-and-1 (=6-and-8), and 7-and-2 (=7-and-9). The other seven string-pairs comprise two strings whose numerals differ by 3-mod-7: 1-and-4, 2-and-5, 3-and-6, 4-and-7, 5-and-1 (=5-and-8), 6-and-2 (=6-and-9), and 7-and-3 (=7-and-10). The initial portion of the analysis and comparison that follow are framed in terms of the 14 string-pairs and disregard the specific sizes of the intervals that would result from the string-pairs. This initial portion of the analysis and comparison can be considered as being based on the conclusion that the tuning of Mesopotamian music is 'heptachordal': in this instance, a cycle of strings that is replicated at every eighth string, i.e., at each 'octave.'

HEPTACHORDAL FRAMEWORK

One might presume that the specific size of the modular, octave interval of the Mesopotamian heptachordal tuning corresponds to the fundamental-frequency ratio 2:1 as in several other cultural settings. However, the analysis and comparison that follows does not assume, or even hypothesize, that the 2:1 ratio is relevant to the Mesopotamian octave. As Anne Kilmer (2000, 114) has pointed out, there is no known term for the octave in Mesopotamian languages. Further, as Gurney (1994, 106) and Martin Litchfield West (1994, 164) have emphasized, the earliest known numerical formulation of musical intervals, i.e., Fragment 6a by Philolaus (ca. 450-400 BCE), was recorded a millennium after the Hurrian scores were notated, though this formulation might have been transmitted to Philolaus from Pythagoras's generation via Hippasus: see, e.g., Huffman (1993, 147-48). Instead, one can conclude that the interval between a particular string and the seventh string above or below it is modular from a passage in the decisive source for our detailed knowledge of Mesopotamian tuning, namely, U.7/80 (ca. 1850 BCE: Gurney 1968, 229-32; Kilmer 1971, 140).

Line ten of U.7/80, which is considered in greater detail below, specifies that at a particular stage in re-tuning a harp or lyre both the second string and the ninth string are altered in a certain way. Like the discursive structure of the other Mesopotamian sources immediately relevant to deciphering the notation of the Hurrian scores, the discursive structure of U.7/80 is paradigmatic. That is, U.7/80, like the other sources directly germane to the scores' notation, is framed in terms of parallel components that can be readily extended beyond the portions that survive. Because of this paradigmatic structure, one can with considerable confidence regard the numerals from 1 to 7 as representing string-classes, rather than merely individual strings. Hence, the pairs of numerals that correspond to the string-pair names employed in the Hurrian scores can be considered to correspond to string-class pairs: e.g., 2-and-5 could have been realized not only by the second and fifth strings of a seven-string harp or lyre but also by the second, fifth, and ninth strings or by the fifth and ninth strings of a harp or lyre comprising nine or more strings; or by the second, fifth, ninth, and twelfth strings, the second, ninth, and twelfth strings, the second, fifth, and twelfth strings, or the second, ninth, and twelfth strings, or the ninth and twelfth strings of a lyre or harp consisting of twelve or more strings; and so forth. As well, the Hurrian scores might have been realized by two or more instruments and/or voices.

In any event, the following preliminary analysis shows that substantial regularities within h.6 and among all 35 Hurrian scores can be discerned solely in terms of pairs of mod-7 string-classes rather than two or more strings in one or more particular registers. All the same, unless otherwise indicated, or for clarity, the rest of this study refers to string-classes as 'strings' and pairs of string-classes as 'string pairs' in order to facilitate reading.

NOTATION OF THE HURRIAN SCORES

Several aspects of h.6's structure can be expressed solely in terms of a) pairs of numbers that correspond to pairs of strings on a harp or lyre and b) the temporal ordering of these numbered string-pairs. Moreover, aspects of the structure of temporally ordered string-pairs are isomorphic with aspects of the structure of the temporally ordered pairs of pitched tones that resulted from them and which are the focus of analysis later in this report.

The Mesopotamian convention for inscribing cuneiform tablets is the basis of our knowledge of the temporal ordering of string-pairs. From the third millennium onward, cuneiform tablets were inscribed from left to right in rows that were arranged in columns from top to bottom. The Hurrian scores were notated by means of names (e.g., Akkadian, *ishartum*: 'upright') that specified particular pairs of named strings (e.g., 'second string' and 'fourth-behind string'). As mentioned above, these specifications appear in CBS 10996, where names of particular string-pairs are aligned with pairs of names for individual strings and with pairs of natural, counting numbers from 1 to 7.

As also mentioned above, another cuneiform tablet, U.3011, lists names for nine individual strings on a musical instrument. That the instrument comprises nine strings is indicated at the end of U.3011 by the phrase 'nine strings,' which serves as a colophon for the preceding nine rows, each of which aligns a Sumerian string-name with its Akkadian counterpart. Only three kinds of stringed instrument are known to have been employed in Mesopotamian music: harps, lyres, and lutes (Dumbrill 2005, 179-344). Because Mesopotamian lutes are known to have had only three strings and Mesopotamian harps and lyres are known to have had as many as nine, or even, more strings, one can conclude that the instrument to which U.3011 refers is a harp or lyre rather than a lute or some other kind of stringed instrument (e.g., a zither, as in ancient China: Thrasher 2000, 1-23).

The precise significance of the string-pair names in CBS 10996 (e.g., 'upright') is still far from clear. Nonetheless, the numbers that CBS 10996 aligns with string-pair names and with pairs of string-names suffice for much of the present study. In this regard, it must be emphasized that CBS 10996's alignment of two string names and two numerals with each string-pair name does not indicate any particular temporal ordering of the tones that would be produced by such a pair of strings.

Some modern transcriptions of h.6 have assumed that the order in which CBS 10996 lists the two string-names and the two numbers that are aligned with each string-pair name implies that the first listed string was realized before the second (e.g., Wulstan 1971, 379-80; Vitale 1982, 261-63; Dumbrill 2005, 130). Other modern transcriptions of h.6 have assumed that both of the strings for which CBS provides names and numbers were performed simultaneously (e.g., Kilmer 1974; West 1994, 177). However, CBS 10996 merely employs the word 'and' (Akkadian: ù), as in the following transliteration and translation of line 13 (Kilmer 1971, 132-33):

transliteration: sa sha-ge$_6$ ù sa 4 uḫri 2 6 sa ishartum

translation: string second and string fourth-behind 2 6 string[-pair] upright

In contrast, such a word as '*lama*' (before: Gelb 1956-2006, vol. 9, 53) could have indicated that string 1 (SA qud-mu-ú) was to be plucked prior to string 5 (SA 5-shú), and such a word as '*ina*' (which often means 'during,' 'while,' or 'when': Gelb 1956-2006, *passim*) could have indicated that the two strings were to sound at the same time. In any event, whatever the temporal order of strings within a string-pair, both tones will have been sounded before those of next string-pair are played.

Most important for the present study, structural regularities in h.6 are consistent with the seven string-numbers being construed not only as natural numbers from 1 to 7 but also as natural numbers mod-7. That the ninth string listed in U.3011 is to be considered to be the same mod-7 as the second string is, as mentioned above, corroborated directly by U.7/80, which is considered in more detail below. That the eighth string listed in U.3011 is to be considered the same mod-7 as the first string is corroborated indirectly in the same source by virtue of U.7/80's paradigmatic structure. Whereas U.7/80 is not discussed in detail until much later in the present study, one can observe meanwhile that the paradigmatic listing of string-pair names in CBS 10996 is also consistent with a mod-7 framework.

CBS 10996 lists 14 string-pair names. The string-pair numbers that are aligned with the first, third, fifth, … and 13th (i.e., the odd-numbered) string-pair names in CBS 10996's listing all differ by 3-mod-7. In contrast, the string-pair numbers that are aligned with the even-numbered string-pair names (i.e., the second, fourth, sixth, … and fourteenth string-pair names in CBS 10996's listing) all differ by 2-mod-7. Moreover, in both interlaced portions of the list, the string-pairs are listed in increments of 1-mod-7: 51, 57; 62, 61; 73, 72; … 47, 46.

Unless otherwise indicated below, string-pair numbers are represented in the rest of this study by two-digit numbers in which the second digit is either 2- or 3-mod-7 greater than the first, e.g., 51 (cf. 58, where 8-5 = 3) rather than 15 (which would correspond to 5-1 = 4), and 57 (cf. 7-5 = 2) rather than 75 (which would correspond to 5-7 = -2 = 5-

mod-7). That such string-pair numbers as 51, 62, and 73 all comprise the same difference between their constituent string-numbers is corroborated further by U.7/80, as discussed below.

PRELIMINARY ANALYSIS OF STRING-PAIRS

In the following preliminary analysis of the Hurrian scores, aspects of h.6's structure are construed in terms of relationships of sameness, adjacency and analogy between successive strings and string-pairs. Prior to the discussion of such relationships, aspects of the Hurrian scores, both temporal and non-temporal, are analyzed statistically.

Statistical Analysis Of Non-Temporal Aspects Of The Hurrian Scores

The present study regards the surviving Hurrian scores as a random sample of the original Hurrian scores. More precisely, a working null hypothesis of the present study is that the Hurrian scores constitute a random sample of strings and string-pairs employed in the larger repertoire of Hurrian music, which, to judge from the fragmentary state of 34 of the scores (see, e.g., Figure. 1, above), must have comprised many more instances of string-pairs. Moreover, disconfirming or not disconfirming various versions of this null hypothesis serves to identify particular aspects of the scores that are idiomatic, i.e., characteristic of all 35 scores, or features of certain scores: in particular, the three scores identified with the *nitkibli* string-pair, namely, h.7 and h.12 as well as h.6, which, as mentioned above, is the only score that survives in continuously notated form. In the first portion of the statistical analysis that follows, temporal relationships between string-pairs are ignored. In this sense, the initial portion of the analysis focuses on non-temporal aspects of the Hurrian scores. Figure 2 presents all 35 scores, with the successive string-pairs represented by means of the 2-digit mod-7 format described above.[2] To facilitate comprehension of the cumulative results, the main conclusions are highlighted in ***bold italic*** typeface.

Frequencies Of Individual Strings

The score of h.6 indicates that 34 string-pairs are to be realized successively. Since each string-pair comprises two numbered strings, there are 68 instances of individual string-numbers to be realized in h.6. If each of the seven numbered strings were realized equally often, each would be realized $68/7 \approx 9.71$ times. However, as Figure 3 shows, string 1 is actually realized six times, string 2 twelve times, and so forth. To what extent are such differences between the ideal, 'expected' frequencies and the actual, 'observed' frequencies probable?

As is usual in such fields as biology as well as in the social and behavioral sciences, the benchmark probability for the present study is .05, which indicates that the probability that the differences between the observed frequencies and the expected frequencies arose merely by chance is 1 in 20. Smaller probabilities than .05 are regarded

[2] John Huehnergard (1997, 563-76) provides a standard listing of cuneiform signs photographed, transcribed, and transliterated in such modern editions. Paired numerals from 1 to 7 represent pairs of strings mod-7: e.g., '25' stands for the string-pair comprising strings (i.e., string-classes) 2 and 5. 3-mod-7 string-pairs are bold and underlined. Hyphens (-) indicate discontinuities in the scores. Asterisks (*) indicate pieces whose colophons unambiguously identify *nitkibli* as the tuning. Laroche (1968, 486) reads string-pair 14 as part of the colophons for h.26 and y, where I have, respectively, 14 and 72 as part of the scores; West (1994, 170, n. 22), seemingly referring to Laroche's list, says that *nitkibli* tuning is 'discernible in four or five cases.' Single parentheses enclose string-pairs whose reading is based on the absence of immediate repetitions among the unparenthesized string-pairs (cf. West 1994, 172). Double parentheses surround a string-pair whose reading is based on the absence, again among the unparenthesized string-pairs, of immediate successions of thirds that are more than a step apart, a regularity discussed in the main body of the present study.

Figure 2. Legible string-pairs in the 35 Hurrian scores: h.6 is based on the revised transliteration of Manfried Dietrich and Oswald Loretz (1975), the remaining 34 are based on the diplomatic edition of Laroche (1955, 1968).

scores:	successive string-pairs:
*h.6	**25** 72 **25** 57 **62** 35 46 57 61 72 **73** 61 72 **14** 24 35 46 57 61 72 **14** 57 61 57 61 57 61 72 **36** **25** **36** **25** **36** **25**
*h.7	**25** 72 - 57 61 (57) - **14** 24 (35) - 61 72 **36**
*h.12	61 - **73** 72
h.2	**14** 57 - 24 (35) - 24 (35) - 35 46 - 72 - **25** 13 – **36** - 35 - 46 57 - **25** - **36**
h.3	57 46
h.4	57 - **14** - 57 - 35 46 - 57 - 57
h.5	61 72 - **14** - 24 (35) - 61
h.8	57 46 - 57 46 57 - 72 61 - **14** - 57 - 61 – 72
h.9	**14** 13 24 - 57
h.10	**14** 57 - ((**14**)) 72 - 35 46 – 72 - 24
h.13	24 **36**
h.14	**36** - 72 **14**
h.16	61 - 46 - 72 - **14** - 13 - 35
h.19	13 24 - 46 57 - 72 **73** - 46
h.20	72 - 61 - 72
h.21	72 - 72 - **25** 61 - 61 72 - 61 - **14** 24 35 - 72 61 **73** - 61 **73**
h.22	72 - 57 - 57 - **25** - **25**
h.23	**14** - 57 46 - 57 46 - 72 **14** - 57 46 – **14**
h.25	72
h.26	61 - 24 - **14**
h.28	**25** - 24 - **25** – 57 - **73** 72
h.30	46 - 46 - 24 - **25**
g	**73** - **14**
j	46 - **73**
n	72 - 61 57
p	61 **73**
q	**14** - **14**

Figure 2 (cont'd).

r	72 - 72 - 13
t	61 - 61
w	**62**
x	72 - **25**
y	57 - **25** - 72
bb	35
ff	35 - 35
gg	**14**

Figure 3. Frequencies of individual string-numbers in h. 6, all 35 Hurrian scores, the 32 non-*nitkibli* scores, and the three *nitkbli* scores: chi-squared probability for h.6 is .34 > .05; for all 35 Hurrian scores, .07 > .05; for the 32 non-*nitkibli* scores vs. the three *nitkibli* scores, .71 > .05.

scores:	**string-numbers:** 1	2	3	4	5	6	7
h.6	8	12	6	5	13	12	12
all 35 Hurrian scores	51	63	38	53	61	55	71
32 non-*nitkibli* scores	39	46	29	46	44	39	53
3 *nitkibli* scores	12	17	9	7	17	16	18

as indicating that the differences between the observed frequencies and the expected frequencies are sufficiently large to be considered 'significant.' Stated more carefully, the null hypothesis that the differences between the observed and expected frequencies arose merely by chance is disconfirmed at the .05-level of significance if the probability is smaller than .05. Conversely, if the probability is larger than .05, the null hypothesis is 'not disconfirmed' (rather than, as uncritical usage would have it, 'confirmed' or 'proven': cf., however, Hagel 2005, 317, 329). As Stephen Stigler (2008) has shown, the .05 level has been, from its origin, a merely conventional standard. Nonetheless, the .05 level has the advantage of facilitating comparisons among studies; in contrast, Hagel's (2005, 318) choice of the .10 level would impede such comparisons. In any event, notwithstanding Hagel's (2005, 328-29) statistical account of certain features of 31 Hurrian scores, particular differences are merely significant or non-significant at a particular level, rather than one difference being regarded as 'more significant' than another.

In order to assess how probable it is that the differences between 6, 12, and so forth on one hand, i.e., the observed frequencies with which particular numbered strings are to be realized, and, on the other hand, ~9.71, i.e., the expected frequency with which each numbered string would be realized, one can calculate the value of chi-squared (χ^2). To calculate the value of chi-squared one can a) subtract each observed frequency from the corresponding expected frequency (e.g., 9.71-6 = 3.71, 9.71-12 = -2.29,…), b) square the results of these subtractions (e.g., 3.71^2 ≈ 13.76, , (-2.29)^2 ≈ 5.24,…), c) divide these squared results by the relevant expected frequency (e.g., 13.76/9.71 ≈ 1.42, 5.29/9.71 ≈ 0.54,…), d) sum the results of these divisions (e.g., 1.42+0.54+…), and e) compare this sum with the sum that would result for a particular probability with which the chi-squared curve associates this sum and particular 'degrees of freedom.' In this case, since there are c=7 columns for the seven string-numbers and r=2 rows for the expected and observed frequencies, the degrees of freedom are (c-1)*(r-1) = 6*1 = 6 (Agresti 2002, 11-25).

Such calculations can be readily carried out by means of, e.g., the chitest function in Microsoft® Excel®. As well, in order to correct for discrepancies between the chi-squared curve, which is formulated in terms of continuous, real numbers, and the observed frequencies that are a basis of the assessment and that necessarily consist of discontinuous, natural, counting numbers, i.e., 1, 2, 3…), one can modify the first step of this calculation (a, above) by, e.g., subtracting 0.5 from the absolute value of each difference before squaring the resulting difference: e.g., (|9.71-6|-0.5)^2 = (3.21)^2 ≈ 10.30, (|9.71-12|-0.5)^2 = (1.79)^2 ≈ 3.20..., as is done when applying Yates' correction (Agresti 2002, 103). Such corrections for continuity are especially important in calculating chi-squared values if more than 20% of the expected frequencies are small (e.g., fewer than five) and can be readily included in one's computations by means of such a resource as Richard Lowry's (2001) online chi-squared calculator for one-way chi-squared distributions.

As Figure 3 shows, the total number of string-number realizations in all 35 Hurrian scores is 51+63+38+53+61+55+71=392. If string-number realizations were equally probable, there would be, ideally, 392/7 = 56 realizations of each string-number. The frequencies with which string-numbers 1, 2, 3, … and 7 are actually realized in all 35 Hurrian scores are, respectively, 51, 63, 38, … and 71. Since the chi-squared probability that such a distribution arose merely by chance is .07, i.e., greater than .05, one can conclude that the differences between 51, 63, 38, and 71 on one hand and 56, 56, 56, … and 56 on the other hand are not significant. As well, one can determine whether the difference between the actual frequencies of the respective string-number realizations in the three *nitkibli* scores and the remaining 32 'non-*nitkibli*' scores is significant.

In the three *nitkibli* scores the actual frequencies of string-numbers 1, 2, 3, … and 7 are 12, 17, 9, … and 18, and the actual frequencies of these string-numbers in the 32 non-*nitkibli* scores are 39, 46, 29, … and 53. Since the total numbers of string-number instances in the three *nitkibli* and 32 non-*nitkibli* scores are, respectively, 96 and 296, and the total number of instances of string-number 1 in both kinds of scores is 51, and the total number of all instances in both kinds of scores is 392, the basic, uncorrected expected frequencies of string-number 1 in the three *nitkibli* and 32 non-*nitkibli* scores are, respectively, $51*96/392 \approx 12.5$ and $51*296/392 \approx 38.5$, as compared with, respectively, 12 and 39. Employing the online calculator available at Kristopher J. Preacher's (2005-09) website results in a corrected chi-squared probability of $0.71>.05$.

In sum, the frequencies with which the seven individual string-numbers are realized in the 35 Hurrian scores do not differ significantly from what one would expect on the basis of a null hypothesis, namely, that all the string-numbers are equally probable. Moreover, the frequencies with which the seven individual string-numbers are realized in the three *nitkibli* scores do not differ significantly from the frequencies with which they are realized in the other 32 non-*nitkibli* scores. In other words, with regard to individual numbered strings, comparing the three *nitkibli* scores, and the 32 non-*nitkibli* scores is not a comparison of 'apples and oranges'; that is, with respect to the distinction between *nitkibli* and non- *nitkibli* scores, the distribution of individual numbered strings is 'homogeneous.'

Subsequent statistical assessments in the present study proceed along similar lines. First, a tendency among all 35 Hurrian scores is determined. Second, with a view to assessing the extent to which the Hurrian scores are homogeneous with regard to this tendency, the same sort of tendency is assessed in the 32 non-*nitkibli* scores and the three *nitkibli* scores. As well, for illustrative purposes and as a background to the subsequent detailed analysis of the only Hurrian score whose notation is continuous, the frequencies relevant to particular assessments are provided for h.6.

Frequencies Of 2- And 3-Mod-7 String-Pairs

Considered in isolation, the preceding assessment of individual string-number frequencies might lead one to conclude that the Hurrian scores resulted from a random selection of the seven numbered strings, i.e., that each instance of a particular string-number was independent of all the other string-number instances. However, each numbered string is a part of four numbered string-pairs: e.g., any instance of string 1 could, in principle, be a part of 13, 14, 51, or 61. Moreover, the frequencies with which the Hurrian scores specify such numbered string-pairs differ significantly. In particular, 2-mod-7 string-pairs are more frequent than 3-mod-7 string-pairs.

As a null hypothesis one would expect that there would be 196*0.5 = 98 2-mod-7 string-pairs among the 196 string-pairs in all 35 scores and one would expect that the number of 3-mod-7 string-pairs would also be 196*0.5 = 98. Instead, there are 139 2-mod-7 string-pairs and 57 3-mod-7 string-pairs in all 35 scores. According to a binomial test (Agresti 2002, 5-25), which, because it is framed in terms of natural-, counting-number frequencies rather than a continuous, real-number curve, is more precise than either an uncorrected or a corrected chi-squared test, this difference between expected and observed frequencies is significant at the .05 level (binomial probability = .00 <.05, calculated by means of T. Webster West's (n.d.) online binomial test resource—unless otherwise noted, probabilities are rounded to two decimal places in the present study). Further, according to Fisher's exact test, the difference between the distributions of 2- and 3-mod-7 frequencies in the three *nitkibli* scores and the remaining 32 non-*nitkibli* scores is not significant (Figure 4).

Like the chi-squared test, Fisher's exact test determines the probability that differences between, or among, two or more sets of observed frequencies could have arisen by chance (Agresti 2002, 91-97). E.g., in the three *nitkibli* scores there are 32 2-mod-7 string-pairs and 16 3-mod-7 string-pairs, and in the 32 non-*nitkibli* scores there are, respectively, 107 and 41. Employing Fisher's exact test to compare these frequencies results in a probability of .47, which is not significant (.47>.05). Like chi-squared and binomial probabilities, Fisher's exact test can be carried out by means of an online resource, e.g., Shigenobu Aoki's (2002) exact test calculator. For the present study, the results of Aoki's calculator were confirmed by means of SAS 9.2 software (SAS Institute 2008).

Figure 4. Frequencies of 2-mod-7 and 3-mod-7 string-pairs in h.6, all 35 Hurrian scores, the 32 non-*nitkibli* scores, and the three *nitkibli* scores: binomial probability for all 35 Hurrian scores is .00 < .05; for the 32 non-*nitkibli* scores, .00 < .05; for the three *nitkibli* scores, .01 < .05; Fisher's exact probability for the 32 non-*nitkibli* scores and three *nitkibli* scores is .47 > .05.

	numbered string-pairs:	
	2-mod-7	3-mod-7
scores:		
h.6	22	12
all 35 Hurrian scores	139	57
the 32 non-*nitkibli* scores	107	41
the 3 *nitkibli* scores	32	16

Where feasible, Fisher's exact test is employed rather than the chi-squared test, with or without correction, especially if fewer than five (or some would say ten) instances are expected or observed in any category. Unlike the chi-squared test and like the binomial test, Fisher's exact test does not calculate continuous, real-number approximations; instead, it calculates probabilities in the same, natural-number terms as the observed frequencies, i.e., as counting numbers: 1, 2, 3, …). However, Fisher's exact test calculates probabilities solely on the basis of observed frequencies and always involves a comparison of at least two observed distributions. In contrast, a one-way chi-squared test, like a binomial test, can also compare a distribution of observed values with abstract expected values, as in the discussion above where individual strings' and string-pairs' observed frequencies are compared with the hypothetical frequencies of 1/7 and 1/2 the total frequency. As well, for very large frequencies and several columns or rows, the chi-squared test requires much less computing capacity.

In short, whereas the seven numbered strings do not differ significantly in their frequencies in all 35 Hurrian scores, the 2- and 3-mod-7 string-pairs do. In particular, 2-mod-7 string-pairs are more frequent in the 35 Hurrian scores, and there is no significant difference between the relative frequencies of 2- and 3-mod-7 string-pairs in the three *nitkibli* and 32 non-*nitkibli* scores. As with individual numbered strings, the ***tendency of 2-mod-7 string-pairs to be more frequent than 3-mod-7 string-pairs*** is distributed homogeneously between the 32 non-*nitkibli* and three *nitkibli* Hurrian scores.

Differences Among The Frequencies Of 2-Mod-7 String-Pairs

Not only are 2-mod-7 string-pairs significantly more frequent than 3-mod-7 string-pairs; as well, certain 2-mod-7 string-pairs are significantly more frequent than others, even though differences among the frequencies of individual strings are not significant (Figure 5). Further, as Figure 6 shows, 2-mod-7 string-pairs whose first digits are larger tend to be more frequent.

Figure 6(a) displays the straight line that most closely fits the succession of string-pair numbers from 13 to 72 and the frequencies of 2-mod-7 string-pairs in all 35 scores. The positive, upward slope of this regression line indicates that string-pairs whose first digits are larger tend to be more frequent. The relatively large value of r^2, namely, 0.90 relative to a range of possible values between 0.00 (for no correlation) and 1.00 (for perfect positive correlation) or -1.00 (for perfect negative correlation), indicates that there is a relatively good, i.e., close, fit between the regression line and the actual frequencies of the string-pairs (Anon. n.d.), which are numbered, in increments of one, from 1 to 7. In contrast, such best-fitting, regression lines between strings 2 and 8(=1), 3 and 9(=2), … and 7 and 13(=6) have much worse fits: respectively, r^2 = 0.00, 0.09, 0.32, 0.18, 0.02, and 0.04.

As Figure 6 also shows, between strings 1 and 7, the regression lines for the 32 non-*nitkibli* scores and the three *nitkibli* scores have a positive slope and the r^2 values for the three *nitkibli* scores are as large as for the other 32 scores (respectively, 0.82 and 0.83). The closest fitting straight lines between various extremes and the corresponding values of r^2 can be calculated by means of the ‘Chart/XY (Scatter)’ command in the Insert menu

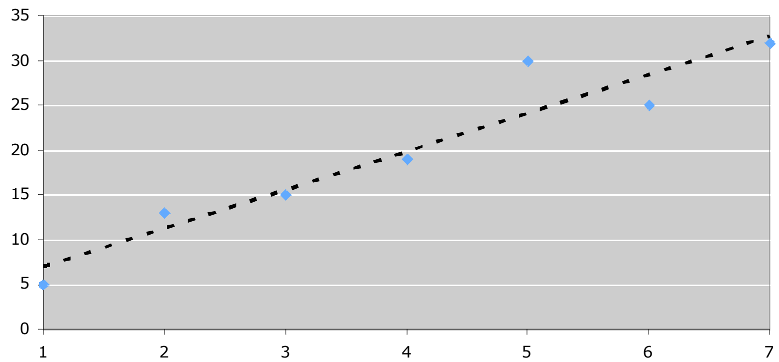
Figure 5. Frequencies of 2-mod-7 string-pairs in h.6, all 35 Hurrian scores, the 32 non-*nitkibli* scores, and the three *nitkibli* scores: chi-squared probability for all 35 Hurrian scores is .00 < .05. Fisher's exact probability for the 32 non-*nitkibli* scores and the three *nitkibli* scores is .47 > .05.

	2-mod-7 string-pairs:						
	13	**24**	**35**	**46**	**57**	**61**	**72**
scores:							
h.6	0	1	2	2	6	6	5
all 35 Hurrian scores	5	13	15	19	30	25	32
the 32 non-*nitkibli* scores	5	11	12	17	22	16	24
the 3 *nitkibli* scores	0	2	3	2	8	9	8

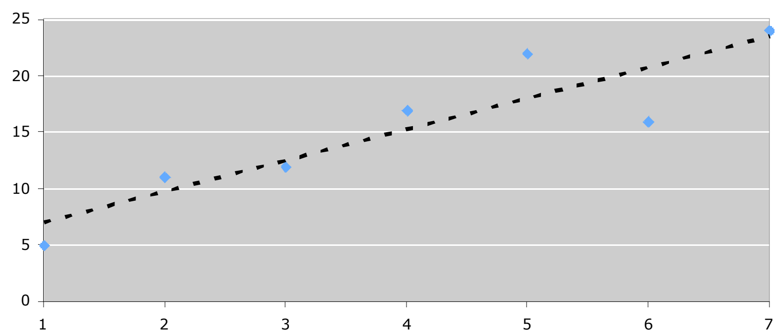
of Microsoft® Excel® followed by the 'Add Trendline…' command in the Chart menu, with 'Linear' as the Trend/Regression type and 'Display R-squared value on chart' as an Option. The hypothesis that all the Hurrian scores were to be performed by a nine-string harp or lyre of the sort whose strings are named in U.3301 (e.g., Hagel 2005, 311-21; cf. also Lawergren and Gurney 1987, 49-51) provides a tentative explanation for this tendency among 2-mod-7 string-pairs. As mentioned above, CBS 10996 aligns the natural numbers from 1 to 7 with the first seven of the nine Sumerian and Akkadian string-names aligned in U.3011. Relative to such a nine-string harp or lyre, the seven 2-mod-7 string-pairs would be arranged from one extreme to the other as follows: 13, 24, 35, 46, 57, 61=68, and 72=79. In this way, the ***homogenous tendency of 2-mod-7 string-pairs with larger first digits to be more frequent*** can be construed as a ***tendency among the nine strings of the harp or lyre whose strings are named in U.3011***, i.e., a tendency that extends from the first to the ninth named string. I return to this tentative explanation below.

Figure 6. Tendency of 2-mod-7 string-pairs (13, 24, …, 61, 72) whose numerically smaller strings (1 of 13, 2 of 24, …, 6 of 61, and 7 of 72) are larger (e.g., 24 as compared with 13, …, 72 as compared with 61) to be more frequent in a) all 35 Hurrian scores, b) the 32 non-*nitkibli* scores, and c) the three *nitkibli* scores (cf. Fig. 5, above). Numerically smaller strings of 2-mod-7 string-pairs (e.g., 1 for 13, 2 for 24) are located along the x-axis; frequencies of respective 2-mod-7 string-pairs are located along the y-axis: the r^2 value for all 35 Hurrian scores is .90; for the 32 non-*nitkibli* scores, .82, and for the three *nitkibli* scores, .83.

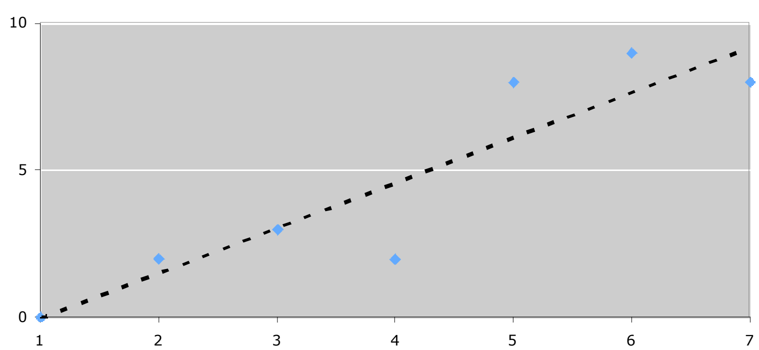
a) all 35 Hurrian scores



b) the 32 non-*nitkibli* scores



c) the 3 nitkibli scores



Differences Among The Frequencies Of 3-Mod-7 String-Pairs

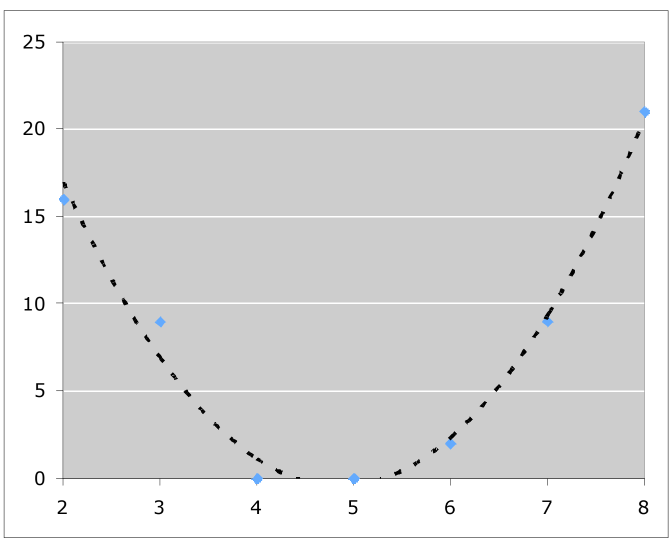
Like the frequencies of the 2-mod-7 string-pairs, the frequencies of the 3-mod-7 string-pairs differ significantly from what one would expect on the basis of a null hypothesis of equal probability for each numbered 3-mod-7 string-pair (Figure 7). Whereas 2-mod-7 string-pairs tend to be more frequent if their first digits are larger and this tendency is well modeled by a straight line, such a rectilinear, straight-line model does not provide as close a fit to the frequencies of the 3-mod-7 string-pairs. If the first digits of 3-mod-7 string-pairs are ordered from string 1 to string 7, the r^2 values for all 35 Hurrian scores, the 32 non-*nitkibli* scores, and the three *nitkibli* scores are, respectively, 0.48, 0.43, and 0.35, and the slopes of the regression lines are uniformly negative. Ordering the first digits of 3-mod-7 string-pairs from 4 to 10 results in the largest r^2 values for rectilinear regressions: respectively, 0.54, 0.37, and 0.82. Whereas this observation might be considered a basis for distinguishing between the *nitkibli* scores and the non-*nitkibli* scores, an alternative approach is to consider the possibility of a non-linear, parabolic model.

A non-linear, parabolic model results in very close fits for such frequencies in all 35 scores, the 32 non-*nitkibli* scores, and the three *nitkibli* scores: respectively, $r^2 = 0.98$, 0.98, and 0.89, for parabolic models in which the first digits of 3-mod-7 string-pairs range from 2 to 1(=8), i.e., from 25 through 36, 47, 51, 62, and 73 to 14 (Figure. 8). Like best-fitting rectilinear models, best-fitting parabolic models can be calculated in Microsoft® Excel®, but with 'Polynomial' and 'Order 2' rather than 'Linear' as the Trend/Regression type.

Figure 7. Frequencies of 3-mod-7 string-pairs in h.6, all 35 scores, the 32 'non-*nitkibli*' scores, and the three *nitkibli* scores: chi-squared probability for all 35 scores is .00 < .05; Fisher's exact probability for the 32 non-*nitkibli* scores and the three *nitkibli* scores is .27 > .05—note that there are only (2-1)*(5-1) = 4 degrees of freedom (df) for Fisher's exact calculation, because there are zero instances of 47 and 51.

	3-mod-7 string-pairs:						
	14	**25**	**36**	**47**	**51**	**62**	**73**
scores:							
h.6	2	5	3	0	0	1	1
all 35 scores	21	16	9	0	0	2	9
the 32 'non-*nitkibli*' scores	18	10	5	0	0	1	7
the 3 *nitkibli* scores	3	6	4	0	0	1	2

Figure 8. Tendency of 3-mod-7 string-pairs to increase in frequency, stepwise away from 47, 51, and 62 (respectively, 4, 5, and 6 on the x-axis) in all 35 pieces (r^2 = .98). Frequencies of individual string-pairs are located along the y-axis: cf. Figure 6, above. The lowest points of the parabolic regression lines for the 32 'non-*nitkibli*' pieces and the three *nitkbli* pieces are similarly between 47 and 62 and have r^2 values of, respectively, .98 and .89 (cf. Figure 7, above).



Common to all groupings of the Hurrian scores are frequencies of zero for string-pairs 47 and 51 and relatively large frequencies for string-pairs 25 and 14. A conjectural aspect of Mesopotamian tuning and the hypothesis that the Hurrian scores were performed on a nine-string harp or lyre can be considered as possible explanations of these features of the way in which 3-mod-7 string-pairs tend to be distributed. In any event, one can observe that, as with individual strings as well as the greater number of 2-mod-7 string-pairs and the tendency of the latter to be more frequent if their first digits are larger, the ***tendency of 3-mod-7 string-pair frequencies to be well modeled by a parabolic curve between 25 and 14 is distributed homogeneously*** in the 35 Hurrian scores. More generally, none of the features of individual numbered strings or string-pairs discussed in the preceding account disconfirms the null hypothesis that ***all 35 Hurrian scores comprise a single idiom, despite the fact that only 3 of the 35 scores are identified with a particular string-pair, namely, nitkibli***.

Statistical Analysis Of Temporal Aspects Of The Hurrian Scores

In the Hurrian scores, there are four kinds of immediate string-pair succession: a) those that comprise two 2-mod-7 string-pairs, b) those that comprise two 3-mod-7 string-pairs, c) those in which a 2-mod-7 string-pair immediately precedes a 3-mod-7 string-pair, and d) those in which a 3-mod-7 string-pair immediately precedes a 2-mod-7 string-pair. One can calculate the expected frequencies of these four kinds of immediate string-pair succession for h.6, all three *nitkibli* scores, and the 32 non-*nitkibli* scores by multiplying, for each of these groups, the number of immediately successive pairs of string-pairs by the relevant ratio(s) of 2-mod-7 and/or 3-mod-7 string-pairs. E.g., among the 33 immediate successions of string-pairs in h.6, the expected frequency of successions in which a 2-mod-7 string-pair is immediately followed by a 3-mod-7 string-pair is 33*(22/34)*(12/34) ≈ 7.54. As Figure 9 shows, ***the actual frequencies of the four kinds of string-pair successions do not differ significantly from what one would expect on the basis of the 2- and 3-mod-7 string-pairs that comprise them*** (one-way chi-squared $p = .79 > .05$; Fisher's exact $p = 0.17 > .05$).

One aspect of immediate string-pair successions in the Hurrian scores does not require statistical analysis. As West (1994, 172) has observed, ***no string-pair is immediately repeated*** in any of the scores. As noted in Figure 2, this uniformity among all 190 string-pair names that are directly legible secures the restoration of five of the partially destroyed string-pair names whose reading would otherwise be uncertain (because certain string-pair names in the Hurrian scores begin or end with the same cuneiform characters). Such restorations not only amplify the basis of statistical

Figure 9. Frequencies of immediate numbered string-pair successions in h.6, all 35 Hurrian scores, the 32 non-*nitkibli* scores, and the three *nitkibli* scores: chi-squared probability for all 35 Hurrian scores is .79 > .05; Fisher's exact probability for the 32 non-*nitkibli* scores and the three *nitkibli* scores is .17 > .05.

	immediate numbered string-pair succession:			
	2-mod-7 to 2-mod-7	**2-mod-7 to 3-mod-7**	**3-mod-7 to 2-mod-7**	**3-mod-7 to 3-mod-7**
scores:				
h.6	16	6	6	5
all 35 Hurrian scores	43	14	17	5
the 32 non-*nitkibli* scores	23	7	8	0
the 3 *nitkibli* scores	20	7	9	5

assessment; they also add weight to a conjecture that two of the scores, namely, the scores of h.6 and h.7, both of which are identified with the *nitkibli* string-pair, convey variants of a single piece.

Immediate Successions Of 2-Mod-7 String-Pairs

Another remarkable aspect of all the immediate 2-mod-7-to-2-mod-7 successions conveyed by the 190+5 = 195 directly legible and restored cuneiform characters referred to above is that ***in each 2-mod-7-to-2-mod-7 succession both strings of a string-pair proceed by a single string***: e.g., in comparison with 61, string-pairs 57 and 72 are one string away whereas 46, 13, 35, and 24 are not. If immediate 2-mod-7-to-2-mod-7 successions were random, only one-third of them would proceed to an adjacent string-pair and the other two-thirds would proceed to string-pairs two or three strings away. It should go without saying that it is extremely improbable that all of the 43 immediate 2-mod-7-to-2-mod-7 successions in the 35 scores would be to an adjacent string-pair (binomial probability < .00 < .05). As Figure 2 indicates, this uniformity among the directly legible and empirically restored string-names is the basis for restoring another partially destroyed string-name.

Almost as remarkable as the uniform succession of 2-mod-7 string-pairs to adjacent 2-mod-7 string-pairs is the observation that ***in immediate successions of adjacent 2-mod-7 string-pairs, the numbered strings of the second string-pair tend to be one string higher than those of the first, rather than one string lower***; i.e., the string-numbers tend to increase by 1-mod-7 from the first to the second of a pair of string-pairs (e.g., from 57 to 61, in contrast to a succession from 57 to 46 or from 61 to 57). If increasing and decreasing string-numbers were equally probable, one would expect 21.5 of each among the 43 immediate successions of this sort in all 35 scores. Instead, 32 of these successions increase by 1-mod-7 and 11 decrease by 1-mod-7. Figure 10 shows that the binomial probability of these increasing successions is .00, i.e., significant at the .05 level. As well, there is no significant difference between the *nitkibli* scores and the non-*nitkibli* scores in this regard (Fisher's exact p=.18). Further, one can interpret the tendency of a 2-mod-7 string-pair to increase, rather than decrease, by 1-mod-7 as a ***temporal asymmetry***, insofar as an event that occurs after another event happens significantly more often in one direction than in the other: in this instance the tendency is toward numerical increase rather than numerical decrease.

Figure 10. Frequencies with which numbered strings in immediate 2-mod-7-to-2-mod-7 successions increase by 1 mod-7 (e.g., 35 to 46 or 57 61) or decrease by 1 mod-7 (e.g. 46 to 35 or 61 to 57) in h.6, all 35 scores, the 32 non-*nitkibli* scores, and the three *nitkibli* scores: the binomial probability for all 35 scores is .00 < .05; Fisher's exact probability for the 32 non-*nitkibli* scores and the three *nitkibli* scores is .18 > .05.

	direction of numbered strings:	
	increase	**decrease**
scores:		
h.6	14	2
all 35 scores	32	11
the 32 non-*nitkibli* scores	15	8
the 3 *nitkibli* scores	17	3

Immediate Successions Of 2- And 3-Mod-7 String-Pairs

Applying to the Hurrian scores an approach developed by Richard Cohn (1996, 15) for mod-12 spaces, one can classify immediate successions that comprise both a 2-mod-7 string-pair and a 3-mod-7 string-pair in terms of the 'smoothest' possible successions between their component strings. Accommodating Cohn's idea to a framework of seven numbered string-classes, one can characterize any pair of consecutive string-pairs in such a smoothest immediate succession in terms of: a) the number of strings the component strings span or traverse; and b) the direction in which they proceed, i.e., whether they numerically increase, decrease, or stay the same (Figure 11).

An outstanding aspect of the Hurrian scores is that ***in the smoothest immediate 2-mod-7-to-3-mod-7 successions and in the smoothest immediate 3-mod-7 to 2-mod-7 successions, at least one string in the first string-pair tends to proceed by 1-mod-7 to a string in the second string-pair***: e.g., in the smoothest succession from 57 to 62, 5 proceeds by 1-mod-7 to 6 whereas 7 proceeds by 2-mod-7 to 2, and in the smoothest succession from 62 to 35, 6 proceeds by 1-mod-7 to 5 and 2 proceeds by 1-mod-7 to 3. As Figure 12 shows, 5/7 of a) the 3-mod-7 string-pairs that can follow a particular 2-mod-7 string-pair plus b) the 2-mod-7 string-pairs that can follow a particular 3-mod-7 string-pair comprise a string that proceeds by 1-mod-7. In contrast, 27 of the 31 immediate 2-to-3- and 3-to-2-mod-7 successions in the 35 Hurrian scores include such a 1-mod-7 succession, as compared with $(5/7)*31 \approx 22.1$. As with other features considered above, this tendency is significant for all 35 scores (binomial $p=.03<.05$).

Moreover, there is no significant difference in this respect between the *nitkibli* scores and the non-*nitkibli* scores (Fisher's exact $p=.10$). Further, as Figure 13 shows, for all 35

Figure 11. Numbers and kinds of progressions of individual strings in smoothest immediate successions between 2-mod-7 and 3-mod-7 string-pairs, illustrated with the 35 string-pair as the first of a pair of immediately successive string-pairs and the 7 possible 3-mod-7 string-pairs as the second. If 35 is the second string-pair of immediate successions that begin with 14, 25, 36, etc., the number of successions in which a numbered string increases or decrease by 1 or 2 from one string-pair to the next is the number of successions in which a numbered string, respectively, decreases or increases by 1 or 2 from one string-pair to the next.

	smoothest progressions from 35 to …							**number of progressions:**
	14	**25**	**36**	**47**	**51**	**62**	**73**	
numbered string…								
increases by 2				5-7			5-7	2
increases by 1			5-6	3-4		5-6		3
remains the same		5-5	3-3		5-5		3-3	4
decreases by 1	5-4	3-2				3-2		3
decreases by 2	3-1				3-1			2

Figure 12. Frequencies with which individual strings in immediate successions from a 2-mod-7 string-pair to a 3-mod-7 string-pair and from a 3-mod-7 string-pair to a 2-mod-7 string proceed by 1-mod-7 strings in h.6, all 35 Hurrian scores, the 32 non-*nitkibli* scores, and the three *nitkibli* scores: binomial probability for all 35 Hurrian scores is .03 < .05; Fisher's exact probability the 32 non-*nitkibli* scores and the three *nitkibli* scores is .10 > .05.

	individual string in smoothest immediate successions …	
	proceeds by 1-mod-7 strings	**does not proceed by 1-mod-7 strings**
scores:		
h.6	9	3
all 35 Hurrian scores	27	4
the 32 non-*nitkibli* scores	15	0
the 3 *nitkibli* scores	12	4

Figure 13. Frequencies with which individual strings in smoothest immediate successions from a 2-mod-7 string-pair to a 3-mod-7 string-pair and from a 3-mod-7 string-pair to a 2-mod-7 string-pair proceed by 1-mod-7 strings or do not proceed by 1-mod-7 strings: for all 35 Hurrian cores, Fisher's exact probability is .61 > .05.

	individual string in smoothest immediate successions …	
	proceeds by 1-mod-7 strings	**does not proceed by 1-mod-7 strings**
from 2-mod-7 to to 3-mod-7	13	1
from 3-mod-7 to 2-mod-7	14	3

Hurrian scores, there is no significant difference in this respect between immediate 2-to-3-mod-7 successions and immediate 3-to-2-mod-7 successions (Fisher's exact p=.61). ***All the immediate 2-mod-7-to-2-mod-7 successions proceed by 1-mod-7, and in a significantly large number of the immediate 2-mod-7-to-3-mod-7 and 3-mod-7-to-2-mod-7 successions at least one of the two strings proceeds by 1-mod-7.*** That a significantly large number of the immediate 2-mod-7-to-2-mod-7 successions increase numerically by 1-mod-7 raises the question of whether one of the two strings in immediate 2-mod-7-to-3-mod-7 and/or 3-mod-7-to-2-mod-7 successions increases numerically by 1-mod-7 in a significantly large number of instances.

There can be immediate successions of 2- and 3-mod-7 string-pairs in which one string increases numerically by 1-mod-7 and the other string decreases numerically by 1-mod-7: e.g., in an immediate succession from 62 to 35, 6 decreases to 5 and 2 increases to 3, and in an immediate succession from 72 to 36, 7 decreases to 6 and 2 increases to 3.

Accordingly, in statistically assessing immediate successions of 2- and 3-mod-7 string-pairs, one compares the frequencies of, e.g., 2-to-3-mod-7 successions in which one string increases numerically by 1-mod-7 with 2-to-3-mod-7 successions in which one string does not increase numerically by 1-mod-7. As Figure 11 (above) shows, in such instances the relevant expected frequency is 3/7 of the 2-to-3-mod-7 successions.

As Figure 14 shows, in immediate 2-to-3-mod-7 successions, one string tends to increase by 1-mod-7. For all 35 scores, the binomial probability is .00 < .05; for the 32 non-*nitkibli* scores and the three *nitkibli* scores, Fisher's exact probability is 1.00 > .05. Conversely, in such successions, one string tends not to decrease by 1-mod-7. For all 35 scores, the binomial probability is .02 < .05; for the 32 non-*nitkibli* scores and three *nitkibli* scores, Fisher's exact probability is .46 > .05 (Figure 15).

Figure 14. Frequencies with which one string-number *increases* by 1-mod-7 in immediate successions *from a 2-mod-7 string-pair to a 3-mod-7 string-pair*: for all 35 Hurrian scores, the binomial probability is .00 < .05; for the 32 non-*nitkibli* acores and the three *nitkibli* scores, Fisher's exact probability is 1.00 > .05.

	one string-number …	
	increases by 1-mod-7 mod-7	**does <u>not</u> increase by 1-**
scores:		
h.6	5	1
all 35 Hurrian scores	13	1
the 32 non-*nitkibli* scores	7	0
the 3 *nitikibli* scores	6	1

Figure 15. Frequencies with which one string-number *decreases* by 1-mod-7 in immediate successions *from a 2-mod-7 string-pair to a 3-mod-7 string-pair*: for all 35 Hurrian scores, the binomial probability is .02 < .05; for the 32 non-*nitkibli* acores and the three *nitkibli* scores, Fisher's exact probability is .46 > .05.

	one string-number …	
	decreases by 1-mod-7	**does not decrease by 1-mod-7**
scores:		
h.6	1	5
all 35 Hurrian scores	2	12
the 32 non-*nitkibli* scores	0	7
the 3 *nitikibli* scores	2	5

In immediate 3-to-2-mod-7 successions, one string does not tend to increase by 1-mod-7. For all 35 scores, the binomial probability is .45 > .05; for the 32 non- *nitkibli* scores and three *nitkibli* scores, Fisher's exact probability is 1.00 > .05 (Figure 16). Moreover, in such successions, one string does not tend to decrease by 1-mod-7. For all 35 scores, the binomial probability is .06 > .05; for the 32 non-*nitkibli* scores and three *nitkibli* scores, Fisher's exact probability is .13 > .05 (Figure 17). In sum, ***immediate 2-mod-7-to-2-mod-7 successions tend to increase numerically by 1-mod-7 and one of the two strings in immediate 2-mod-7-to-3-mod-7 successions tends to increase numerically by 1-mod-7.***

Like the tendency of both strings to increase by 1-mod-7 in 2-mod-7-to-2-mod-7 successions, the tendency for the string-number of one string in a 2-mod-7 string-pair to increase by 1-mod-7 when immediately followed by a 3-mod-7 string-pair can be

Figure 16. Frequencies with which one string-number *increases* by 1-mod-7 in immediate successions *from a 3-mod-7 string-pair to a 2-mod-7 string-pair*: for all 35 Hurrian scores, the binomial probability is .45 > .05; for the 32 non-*nitkibli* acores and the three *nitkibli* scores, Fisher's exact probability is 1.00 > .05.

	one string-number …	
	increases by 1-mod-7	**does not increase by 1-mod-7**
scores:		
h.6	3	3
all 35 Hurrian scores	8	9
the 32 non-*nitkibli* scores	4	4
the 3 *nitikibli* scores	4	5

Figure 17. Frequencies with which one string-number *decreases* by 1-mod-7 in immediate successions *from a 3-mod-7 string-pair to a 2-mod-7 string-pair*: for all 35 Hurrian scores, the binomial probability is .06 > .05; for the 32 non-*nitkibli* acores and the three *nitkibli* scores, Fisher's exact probability is .13 > .05.

	one string-number …	
	decreases by 1-mod-7	**does not decrease by 1-mod-7**
scores:		
h.6	3	3
all 35 Hurrian scores	11	6
the 32 non-*nitkibli* scores	7	1
the 3 *nitikibli* scores	4	5

understood as an instance of ***temporal asymmetry*** in the Hurrian idiom. In both instances, the basis of the asymmetry is both temporal and numerical: in particular, the contrasts between a string-pair that immediately precedes another string-pair and a string-pair that immediately succeeds another string-pair, and between a string-number that increases by 1-mod-7 and a string-number that does not increase by 1-mod-7.

Among the 35 Hurrian scores, there is an even more precise tendency. As noted above, ***in the smoothest immediate 2-to-3-mod-7 successions in the Hurrian scores one string tends to increase by 1-mod-7; as well, the other string tends to increase by 2-mod-7***: specifically, in 9 of 13 instances in the 35 scores. As shown above in Figure 11, for any 2-mod-7 string-pair, there are 3 possible 2-to-3-mod-7 successions in which one string-number increases by 1-mod-7: e.g., the 2-mod-7 string-pair 72 might be immediately followed by 73, 14, or 36. In only one of these three kinds of succession does the other string-number increase by two: e.g., where 72 is followed by 14. Whereas one would expect that one-third of the 35 scores' 13 immediate 2-to-3-mod-7 successions in which one string-number increases by 1-mod-7 would proceed in this way, more than twice as many do so, namely, nine (rather than 13/3 ≈ 4.3), for which the binomial probability is .01<.05. Moreover, the difference between the *nitkibli* and non-*nitkibli* scores is not significant in this regard (Fisher's exact p=.27>.05: Figure 18).

Conversely, ***in a significantly large number of the immediate 3-to-2-mod-7 successions in which one string-number decreases by 1-mod-7, the other string-number does not decrease by 2-mod-7*** (Figure 19). As well, in contrast to immediate 2-to-3-mod-7 successions, there are no instances among the immediate 3-to-2-mod-7 successions in which one string-number increases by 1-mod-7 and the other string-number increases by 2-mod-7, and there are no instances among the 2-to-3-mod-7 successions in which one string-number decreases by 1-mod-7 and the other string-

Figure 18. Frequencies of immediate 2-to-3-mod-7 string-pair successions where one string-number increases by 1-mod-7 and the other string-number increases by 2-mod-7: for all 35 Hurrian scores, the binomial probability is .01 < .05; for the 32 non-nitkibli acores and the three *ni*tkibli scores, Fisher's exact probability is .27 > .05.

	the other string-number …	
	increases by 2-mod-7	**does not increase by 2-mod-7**
scores:		
h.6	3	2
all 35 Hurrian scores	9	4
the 32 non-*nitkibli* scores	6	1
the 3 *nitikibli* scores	3	3

Figure 19. Frequencies of immediate 3-to-2-mod-7 string-pair successions where one string-number decreases by 1-mod-7 and the other string-number decreases by 2-mod-7: for all 35 Hurrian scores, the binomial probability is .01 < .05; for the 32 non-*nitkibli* scores and the three *nitkibli* scores, Fisher's exact probability is 1.00 > .05.

	the other string-number …	
	decreases by 2-mod-7	**does not decrease by 2-mod-7**
scores:		
h.6	1	2
all 35 Hurrian scores	3	8
the 32 non-*nitkibli* scores	2	5
the 3 *nitikibli* scores	1	3

number decreases by 2-mod-7. In each of these ways, then, immediate 2-to-3 string-pair successions convey ***temporal asymmetry.***

Finally, in all of the 35 Hurrian scores, the ***only immediate 2-to-3-mod-7 succession in which one string-number does not increase by 1-mod-7 appears in the immediate succession 72-25, which occurs directly after the outset of h.6*** (i.e., between h.6's second and third string-pairs). Among all 35 scores, this is not only the sole instance of an immediate 2-to-3-mod-7 succession in which one string-number does not increase by 1- mod-7; as well, the 72-25 succession is the only instance where a string-number in an immediate 2-to-3-mod-7 succession decreases by 2-mod-7.

Immediate 3-Mod-7-To-3-Mod-7 Successions

In h.6, all of the immediate 3-mod-7-to-3-mod-7 successions occur at the end: in particular, within the final six string-pairs: 36 25 36 25 36 25. As in all of the immediate 2-mod-7-to-2-mod-7 successions in h.6, ***all of the immediate 3-mod-7-to-3-mod-7 successions in h.6 proceed by a single string-number***.

In contrast to immediate 2-to-2-mod-7, 2-to-3-mod-7, and 3-to-2-mod-7 successions, there are no immediate 3-to-3-mod-7 successions in the other *nitikbli* scores or in the 32 non-*nitkibli* scores (Figure 9, above). This is not entirely surprising as, on the basis of the frequencies with which 3-mod-7 string-pairs occur in all 35 Hurrian scores, only (57/196)*(57/196)*79 ≈ 6.7 3-to-3-mod-7 successions would be expected, i.e., 1.7 beyond those that occur in h.6 (cf. Figures 4 and 9, above). Although one cannot compare the various sorts of 3-to-3-mod-7 successions that might have appeared in Hurrian scores other than h.6, one can observe that ***the only immediate 3-to-3-mod-7 successions, namely those at the end of h.6, oscillate between adjacent strings***.

Three Or More Immediately Successive String-Pairs

The eight possible kinds of immediate successions of three string-pairs can be represented as follows: 222, 223, 232, 322, 233, 323, 332, and 333, where 2 and 3 stand for, respectively, 2- and 3-mod-7 string-pairs. As shown in Figure 4, the empirical probabilities of 2- and 3-mod-7 string-pairs in all 35 Hurrian scores are, respectively, 139/196 ≈ .71 and 57/196 ≈ .29, and provide the expected frequencies for the eight possible kinds of immediately successive string-pair triples which are a basis for statistically assessing the frequencies of the eight kinds of immediate string-pair triples in Figure 20.

Figure 20. Frequencies of the eight kinds of string-pair triples in immediate succession: '2' and '3' represent, respectively, 2-mod-7 and 3-mod-7 string-pairs, so that '223' represents a 2-mod-7 string-pair followed by a 2-mod-7 string-pair followed by a 3-mod-7 string-pair; for all 35 Hurrian scores, the one-way chi-squared probability is .07 > .05, and for the 32 non-*nitkibli* scores and the three *nitkibli* scores Fisher's exact probability is .61 > .05.

	Eight kinds of string-pair triples in immediate succession:							
scores:	**222**	**223**	**232**	**322**	**233**	**323**	**332**	**333**
h.6	12	4	5	4	1	2	0	4
all 35 Hurrian scores	14	6	5	7	1	2	0	4
the 32 non-*nitkibli* scores	1	1	0	2	0	0	0	0
the 3 *nitkibli* scores	13	5	5	5	1	2	0	4

As Figure 20 shows, the difference between the frequencies of the eight kinds of immediate succession and what one would expect on the basis of the relative frequencies of 2- and 3-mod-7 string-pairs in all 35 Hurrian scores is not significant (one-way chi-squared probability = .07 > .05) and the corresponding difference between the 32 non-*nitkibli* scores and the three *nitkibli* scores is not significant (Fisher's exact probability = .61 > .05).

With regard to immediate string-to-string successions, the oscillation of 3-mod-7 string-pairs between adjacent strings at the end of h.6 is similar to the following oscillation of 2-to-2-mod-7 string-pairs between adjacent strings earlier in the same score: 57 61 57 61 57 61. As well, within h.6 there are eight instances where three consecutive 2-mod-7 string-pairs increase numerically by 1-mod-7, but there are no immediate successions in which all three such string-pairs decrease numerically: this contrast is another instance of ***temporal asymmetry***.

In comparison with h.6, the other, discontinuous scores contain relatively few instances of three string-pairs in immediate succession. One of the other two *nitkibli* scores, namely h.7, which might be a variant of h.6, comprises three instances, each of which occurs in h.6:

h.7 57 61 57   **14** 24 35   61 72 **36**

Among three of the remaining 32 non-*nitkibli* scores there are only four instances:

h.8 57 46 57

h.9 **14** 13 24

h.21   **14** 24 35   72 61 **73**

Of these seven instances, two comprise an oscillation between adjacent 2-mod-7 string-pairs. Among these, the succession 57 61 57 in h.7 also appears in h.6. Otherwise, there is little that is remarkable among these seven immediately successive string-pair triples. E.g., within the seven immediate successions of three string-pairs, three of the four immediate 2-to-2-mod-7 successions in h. 7 increase by 1-mod-7, and three of the five in the non-*nitkibli* scores: frequencies that correspond quite closely to the frequencies of isolated, 'un-enchained' 2-to-2-mod-7 successions in all 35 Hurrian scores.

OVERVIEW OF H.6

The following overview summarizes features of h.6 that are frequent or infrequent in the other Hurrian scores. First, the ratio of 2-mod-7 string-pairs to 3-mod-7 string-pairs in h.6 and the other scores is approximately 2:1. Second, as in the other Hurrian scores, all of the immediate successions of 2-mod-7 string-pairs proceed to an adjacent string-pair. Third, also as in the other Hurrian scores, most of the immediate successions of 2-mod-7 string-pairs increase numerically by 1-mod-7. Fourth, in the other Hurrian scores, all of the immediate successions from a 2-mod-7 string-pair to a 3-mod-7 string-pair comprise an individual string that increases numerically by 1-mod-7. As noted above, all but one such succession in h.6 proceeds in this way, the 72-25 succession toward the outset being an anomaly. Fifth, as in the other Hurrian scores, the other numbered string tends to increase numerically by 2-mod-7. Finally, in h.6, immediate successions of three or more consecutive 2-mod-7 string-pairs either increase numerically by 1-mod-7 or oscillate between adjacent string-pairs. In the other scores, there are only two immediate successions of three consecutive 2-mod-7 string-pairs and both oscillate between adjacent string-pairs.

A PRELIMINARY ANALYSIS OF H.6

The preceding account focuses on how frequently strings, kinds of string-pairs, and kinds of immediate string and string-pair successions occur in the extant Hurrian scores, and distinguishes between scores that identify or do not identify the string-pair name *nitkbli* in their colophons. The following analysis focuses instead on relationships between and among strings, string-pairs, etc. that occur within the only Hurrian score which, in its extant form, is continuously notated, namely, h.6.

Relationships among Strings and String-Pairs in H.6

The first part of the following analysis of h.6 focuses on relationships of sameness, adjacency, and analogy. By means of these relationships, the scope of the analysis extends beyond the upper limit of three immediately successive string-pairs posed by the other 34 Hurrian scores, a limitation that constrains the preceding statistical account.

Unlike any of the immediate successions of string-pairs in h.6 (or in any of the Hurrian scores for that matter), the non-immediate successions 25…25 and 14…14 in h.6 constitute repetitions. More precisely, the 25s and 14s within these non-immediate successions are the same not only with regard to the number of strings they span, namely, 3-mod-7; as well, they are the same with regard to the individual strings they comprise: 2 and 5, and 1 and 4. As such, the string-pairs in these non-immediate successions are 'more similar' to each other than the string-pairs in other non-immediate successions of

3-mod-7 string-pairs (e.g., 25...62 and 62…73), for they are the same not only with regard to the number of strings they span but also with regard to both of their constituent strings.

One can specify the sense in which 25…25 and 14…14 constitute non-immediate successions in terms of both sameness and adjacency. The second 25 string-pair is not only the same as the first; as well, the second 25 string-pair is the first 3-mod-7 string-pair that follows the first 25 string-pair. Similarly, the second 14 string-pair is the first 3-mod-7 string-pair that follows the first 14 string-pair. That is, among 3-mod-7 string-pairs, the 25 string-pairs are in immediate succession and the 14 string-pairs are in immediate succession. In other words, among 3-mod-7 string-pairs, the 25 string-pairs and the 14 string-pairs are temporally adjacent.

Somewhat less similar to one another are the string-pairs in the succession 25…62. Both of the string-pairs in this succession are 3-mod-7 string-pairs and they have one string in common, namely, string 2. Unlike 25…25 and 14…14, one of the strings in each string-pair is not the same as one of the strings in the other, insofar as string 5 differs from string 6. Accordingly, the string-pairs in the successions 25…25 and 14…14 are 'the same' to a 'greater extent,' that is, in more ways, than those in the succession 25…62. More precisely, there is no way in which the string-pairs in 25…62 are the same that is not also a way in which the string-pairs in 25…25 and 14…14 are the same, but the converse does not hold.

Determining degrees or gradations of similarity can be problematic if, among several things, the ways, or respects, in which pairs of things are the same differ, or merely overlap, as discussed by, e.g., Rahn (1982, 3-4). However, in the present case, degrees of similarity can be determined unambiguously, for the pairs of paired things just considered are the same in all but one way.

Relationships of Sameness and Adjacency in Pairs of String-Pairs

As understood here, a relationship of sameness is both i) reflexive and ii) symmetric insofar as i) it holds between two things and themselves (i.e., severally), and ii.a) it holds between one of the things and the other thing (again, severally) and ii.b) it holds between the other thing and that thing (yet again, severally: cf., e.g., Rahn 1992, 162-68; 1994, 1.0-1.6). E.g., any string-pair that spans the same number of strings mod-7 as another string-pair also spans the same number of strings mod-7 as itself, and any string in a string-pair that is the same as a string in another string-pair is also the same as itself. For clarity:

$(x)(y)(xS_{R}y \Leftrightarrow (xRx\ .\ yRy)\ .\ (xRy\ .\ yRx))$

For any things, x and y,

x is, with regard to relationship R, the same as y if and only if

i.a) x is related, by relationship R, to x, and

i.b) y is related, by relationship R, to y, and

ii.a) x is related, by relationship R, to y, and

ii.b) y is related, by relationship R, to x.

(Willard Van Orman Quine's (1966) *Elementary Logic* provides an introduction to the notational conventions of first-order predicate logic employed here.)

In specifying that the sameness of the number of strings mod-7 that string-pairs span and that the sameness of one or both strings in string-pairs are instances of sameness serves analytically as a basis for gathering or grouping individual strings and individual string-pairs without assuming more than what has already been concluded about numbered strings and string-pairs and their realization in the Hurrian scores. Similarly, no additional assumptions are involved in positing degrees of sameness as discussed above. For further clarity:

$(x)(y)(z)(xS_{RR'}GEy,z \Leftrightarrow (xS_{R}y\ .\ xS_{R'}y)\ .\ –(xS_{R}z\ .\ xS_{R'}z))$

For any things, x, y, and z,

x is, with regard to respects R and R', the same as y to a greater extent than x is the same as z if and only if

i.a) x is the same as y in a particular respect, R,

i.b) x is the same as y in another respect, R', and

ii) x is not the same as z both in a) respect R and b) respect R'.

In general, one thing immediately precedes another thing if it precedes the other thing and there is no other thing that it precedes that also precedes the other thing. For clarity,

$(x)(y)(xIPy \Leftrightarrow xPy\ .\ –(\exists z)(xPz\ .\ zPy))$

For any things, x and y,

x immediately precedes y if and only if

i) x precedes y, and

ii) there is no thing, z, such that

a) x precedes z, and

b) z precedes y.

One can formulate the special sense in which the first 25 string-pair immediately precedes the second 25 string-pair, and similarly for 14…14, as follows:

$(x)(y)(xIP_{3\text{-mod-}7}y \Leftrightarrow (3\text{-mod}7x\ .\ 3\text{-mod-}7y)\ .\ xPy\ .\ –(\exists z)(3\text{mod-}7z\ .\ xPz\ .\ zPy))$

For any things, x and y,

x immediately precedes 3-mod-7, i.e., in a 3-mod-7 manner, y if and only if

i) x is 3-mod-7 and y is 3-mod-7, and

ii) x precedes y, and

iii) there is no thing, z, such that

a. z is 3-mod-7, and

b. x precedes z, and z precedes y.

In contrast, the first 25 string-pair immediately precedes the 72 string-pair in the usual sense. Just as the first 25 is the same as the second 25 to a greater extent than either is the same as 62, the first 25 immediately precedes 72 to a greater extent, i.e., 'more immediately,' than the first 25 precedes the second 25.

Analogy

Even less sameness obtains between the string-pairs in the successions 62…73 and 73…14 than in the succession 25…62. In these, both string-pairs span the same number of strings, namely, 3-mod-7, but they are not the same with regard to any of their constituent strings. Notwithstanding their lack of 'common' strings, the string-pairs in these numerically increasing successions are linked by relationships of analogy.

In 62…73, string 6 is 3-mod-7 less than string 2 and string 7 is 3-mod-7 less than string 3. Expressed as an analogy, 6:2::7:3, read '6 is to 2 as 7 is to 3.' Similarly, string 6 is 1-mod-7 less than string 7 and string 2 is 1-mod-7 less than string 3, or, analogically, 6:7::2:3. Moreover, the same sorts of paired analogical relationships hold in the succession 73…14: specifically, 7:3::1:4 and 7:1::3:4.

In an analogical relationship, as understood here, a) one part of one thing is related to another part of that thing in the same way as one part of another thing is related to another part of that other thing, and b) one part of one thing is related to one part of another thing in the same way as another part of the one thing is related to another part of the other thing. For clarity:

$$(x')(x'')(y')(y'')((x'+x''\ A\ y'+y'') \Leftrightarrow ((x'Rx''\ .\ y'Ry'')\ .\ (x'R'y'\ .\ x''R'y'')))$$

For any things, x', x'', y', and y'', the sum of x' and x'' (i.e., the thing that comprises all and only x' and x'') is analogous to the sum of y' and y'' if and only if

i) x' is related in a particular way, R, to x'' and y' is related in the same way, R, to y'', and

ii) x' is related in a (possibly contrasting) way, R', to y' and x'' is related in the same (possibly contrasting) way, R', to y''.

(Nelson Goodman (1966, 46-56) provides a formulation of things, more precisely, 'individuals,' as parts or sums of things on the basis of the 'overlaps' predicate, which avoids ontological difficulties that arise in formulations based on sets, for which see Quine (1969, 1-2).)

As with sameness relationships, the specification that the pair-wise sameness of the number of strings mod-7 that string-pairs span is an instance of analogy serves to characterize in a more general way pairs of individual string-pairs without assuming more than what has already been concluded about numbered strings and string-pairs and their realization in the Hurrian scores. As well, the present study's sameness, adjacency, and analogy relationships can be considered restrained instances of David Lewin's (1987, xi) highly general notion of 'the interval from s to t' in the sense of 'int(s,t)', Adam Ockelford's (2005, 18-29) 'zygonic' relationships, and Rahn's (1983, 49-52; 1985) notion of 'relational richness'). Moreover, sameness relationships can be understood as instances of analogical relationships.

For instance, in the succession 25…62, 2:5::6:2 and 2:6::5:2, the analogy involves relationships between strings of different string-pairs; in 25…25, 2:5::2:5 and 2:2::5:5, the analogy involves relationships between strings of different string-pairs but would also hold for relationships between the strings of a single string-pair, for in the preceding formulation of analogy there is no specification that x'≠x'', y'≠y'', x'≠y', or x''≠y''; i.e., there is no specification that x' is not precisely the same thing, e.g., precisely the same instance or realization of a particular string, as x'', or that y' is not precisely the same thing, e.g., precisely the same instance or realization of a particular string, as y'', etc.

Such analogical relationships also hold among the strings of immediately successive string-pairs. Between immediately successive string-pairs, the sameness relationships that combine to constitute relationships of analogy are amplified by their immediacy. 62 immediately precedes 73 among 3-mod-7 string-pairs but not among string-pairs in general, i.e., not among both 3-mod-7 and 2-mod-7 string-pairs. In contrast, 24 immediately precedes 35 not only as far as 2-mod-7 string-pairs are concerned, but more generally, as far as all string-pairs are concerned.

As noted above, the two instances of 25 at the outset of h.6 are 'the same' to a greater extent than are the second instance of 25 and the next 3-mod-7 string-pair, namely, 62.

The two instances of 25 at the outset of h.6 are the same not only with regard to the number of strings they span but also with regard to both of their component strings, whereas the second instance of 25 and the 62 string-pair that follows it are the same with regard to the number of strings they span but not with regard to both of their component strings. Similarly, 24 and 35 are adjacent to a greater extent than 62 and 73, for the latter are immediately successive only as 3-mod-7 string-pairs whereas 24 and 35 are immediately successive both as 2-mod-7 string-pairs and as string-pairs in general.

String and string-pairs can be adjacent not only with regard to time; as well, individual strings can be adjacent with regard to their mod-7 numbering. E.g., in 62…73 the strings that constitute 62 are, severally, 1-mod-7 smaller than the respective strings that constitute 73. In other words, 6 is numerically just smaller than 7, and 2 is numerically just smaller than 3. Similarly, in 24…35, the strings of 24 are numerically just smaller than the strings of 35.

Pairs Of Paired String-Pairs

Within 25…25, the first string-pair is analogous to the second string-pair; similarly, for both of the string-pairs in 14…14. Moreover, 25…25 as a whole is analogous to 14…14 as a whole. To distinguish the analogous relationships between 25 and 25 and between 14 and 14 from the analogous relationship between 25…25 and 14…14, one can define the latter as a doubly analogous relationship. In general:

(x)(x')(y)(y')(x+x'DAy+y' ⇔ xAx' . yAy')

For any things, x, x', y, and y',

the sum of x and x' is doubly analogous to the sum of y and y' if and only if

i) x is analogous to x', and

ii) y is analogous, i.e., in the same way, to y'.

Similarly, in the succession 36 25 36 25 36 25 at the end of h.6, immediately successive instances of 36 and 25 are analogous: 36 A 25, 36 A 25, and 36 A 25. As well, each instance of the immediately successive pair of string-pairs 36-25 is doubly analogous: 36-25 DA 36-25 and 36-25 DA 36-25. Moreover, at the beginning of the 2-mod-7 succession 57 61 57 61 57 61 72, 57-61 DA 57-61 and 57-61 DA 57-61.

Transitive Relationships Among String-Pairs

In the succession 62…73…14, 62:73::73:14. That is, the second of one analogical pair is the first of the next analogical pair. Numerical metaphors include the arithmetic series 1, 2, 3, 4,…, where 1:2::2:3, 2:3::3:4, etc., and the geometric series 1, 2, 4, 8,…, where 1:2::2:4, 2:4::4:8, etc. In such transitively related successions, analogical relationships are iterated. For clarity:

(x)(y)(z)(xTAy,z ⇔ xAy . yAz)

For any things, x, y, and z, x is transitively analogous to y and z if and only if

i) x is analogous to y, and

ii) y is analogous, i.e., in the same ways, to z.

Such transitively analogous relationships hold also among several immediate successions of 2-mod-7 string-pairs: 35 A 46 A 57 A 61 A 72; 24 A 35 A 46 A 57 A 61 A 72; 57 A 61 A 72.

Not only is each immediately successive pair of immediately successive 36-25 string-pairs doubly analogous (36-25 DA 36-25 and 36-25 DA 36-25); as well, the entire succession of immediately successive 36-25 string-pairs is transitively analogous: 36-25 DA 36-25 DA 36-25. Similarly, the beginning of the succession 57 61 57 61 57 61 72 is transitively analogous: 57-61 DA 57-61 DA 57-61. Moreover, as mentioned above, the last pair of paired string-pairs overlaps the transitively analogous succession 57 A 61 A 72.

In sum, h.6 can be interpreted as a network of relationships. Broadly, these include relationships of sameness, adjacency and analogy. Whereas the analogical relationships discussed above are framed in terms of sameness and adjacency relationships, the latter can be expressed in terms of a single predicate for time and a single predicate for strings, neither of which presumes more than what the Hurrian scores convey through the Mesopotamian scribal practice of ordering words and numerals from left to right in rows and from top to bottom in columns and from the concordance of terms in the Hurrian scores with their counterparts in other cuneiform tablets.

A Parsimonious Basis For Temporal Relationships

With regard to time, the relationship whereby one thing, e.g., an individual string or string-pair, is considered to precede another thing, e.g., another string or string-pair, can be defined in terms of the predicate 'is at least as early as' in the following manner:

$(x)(y)(xPy \Leftrightarrow xAEy\ .\ -yAEx)$

For any things, x and y, x precedes y if and only if

i) x is at least as early as y, and

ii) y is not at least as early as x.

As shown above, the relationship whereby a thing immediately precedes another thing can be defined in terms of the 'precedes' predicate. As well, the relationship whereby two things merely differ temporally can be defined as follows:

(x)(y)(xDTy ⇔ xPy v yPx . –(xPy . yPx))

For any things, x and y, x differs in time from y if and only if

i) x precedes y or y precedes x, and

ii) it is not true that both

a) x precedes y, and

b) y precedes x.

Such a relationship of temporal difference holds among the string-pairs treated statistically in the preliminary analysis above.

Since the 'precedes' predicate is defined as asymmetric, it is also irreflexive; that is, the 'precedes' predicate does not hold between any thing and itself. As a consequence, one thing necessarily precedes, immediately precedes, or differs in time from, another thing; that is, in the formulations above, x≠y, i.e., x is not precisely the same thing as y. Further, the 'precedes' and 'immediately precedes' predicates are necessarily asymmetric, whereas the 'differs in time from' predicate is symmetric. This contrast between such symmetric and asymmetric predicates is a basis for the distinction between temporal asymmetry and what might be termed 'atemporality' or 'semi-temporality' in the statistical account of the Hurrian scores and in the analysis of h.6.

The 'is at least as early as' predicate also serves to distinguish relationships within string-pairs from relationships between string-pairs. In general, 'is simultaneous with' can be defined as follows:

$(x)(y)(xSWy \Leftrightarrow xAEy\ .\ yAEx)$

For any things, x and y, is simultaneous with y if and only if

i) x is at least as early as y, and

ii) y is at least as early as x.

The ‘is simultaneous with’ predicate is both a) symmetric and b) reflexive in the sense that if it holds between one thing and another, possibly different, thing, a) the reverse is also true, and b) it necessarily holds between those things and themselves:

$(x)(y)(xSWy \Rightarrow ySWx)$

For any things, x and y, if x is simultaneous with y, then y is simultaneous with x.

$(x)(y)(xSWy \Rightarrow xSWx)$

For any things, x and y, if x is simultaneous with y, then x is simultaneous with x.

In the Hurrian scores, a string-pair is simultaneous with itself, a string is simultaneous with itself, a string that is part of a string-pair is simultaneous with that string-pair, that string-pair is simultaneous with that string, and both strings of that string-pair are simultaneous with each other. Whether such strings were actually realized in a precisely simultaneous manner is not certain, for, as discussed above, Mesopotamian notation specifies simultaneity and succession only within the framework of individual string-pairs; that is, the individual string-pair is the finest level of temporal ‘grain’ within the scores. In this sense of simultaneity, string-pairs can be defined as follows:

$(x)(SPx \Leftrightarrow -(\exists x')(x'<x\ .\ –x'SWx))$

For any thing, x, x is a string-pair if and only if

i) there is no thing, x', such that

a) x' is a part of x, and

b) x' is not simultaneous with x.

As just shown, one can define such temporal relationships as ‘precedes,’ ‘immediately precedes,’ and ‘differs in time from’ in terms of the single asymmetric predicate ‘is at least as early as.’ As also discussed, two kinds of things that might be described by the English-language phrase ‘immediately precedes’ can be distinguished on the basis of whether the things to which it refers involve string-pairs in general, or more precisely string-pairs of a certain sort, i.e., 2-mod-7 string-pairs or 3-mod-7 string-pairs.

One need not consider the relationship of sameness that holds between 2-mod-7 string-pairs as an undefined relationship and the relationship of sameness that holds between 3-mod-7 string-pairs as another, distinct undefined relationship. Instead, one can define both in terms of the predicate ‘is at least as much larger, mod-7, than … as … is than…, also mod-7.’ E.g., a 2-mod-7 string-pair is at least as much larger, mod-7, than an individual string as a 3-mod-7 string-pair is, mod-7, than a 2-mod-7 string-pair, but a 3-mod-7 string-pair is not at least as much larger, mod-7, than a 2-mod-7 string-pair as a 2-mod-7 string-pair is, mod-7, than an individual string. In other words, a 2-mod-7 string-pair exceeds an individual string to a greater extent than a 3-mod-7 string-pair exceeds a 2-mod-7 string-pair.

More formally:

$(x)(Sx \Leftrightarrow (\exists y)(SPy\ .\ x<y\ .\ –(\exists x')(x'<x\ .\ xALMLT_{mod\text{-}7}x',x,x))$

For any thing, x, x is an individual string if and only if there is at least one thing, y, such that

i) y is a string-pair,

ii) x is a part of y, and

iii) there is no thing, x', such that

a) x' is a part of x, and

b) x is, mod-7, at least as much larger than x' as x is, mod-7, than x.

Here one can note that the predicate S, i.e., 'is a string-pair, is defined above in terms of the general predicate 'is a part of' and the predicate SW 'is simultaneous with.'

$(x)(0\text{-mod-}7x \Leftrightarrow (\exists y)(Sy\ .\ yALMLT_{mod\text{-}7}x,y,y))$

For any thing, x, x is 0-mod-7 if and only if there is at least one thing, y, such that

i) y is an individual string, and

ii) y is at least as much larger, mod-7, than x, as y is, mod-7, than y.

Whereas the predicate S specifies an individual string, i.e., a particular instance or realization of a particular string, the predicate 0-mod-7 specifies the string-intervallic size or magnitude of such an individual string. In this way, one can specify sameness relationships between strings that are parts of different string-pairs or that are different parts of a single string-pair.

$$(x)(y)(2\text{-mod-}7x\ .\ 1\text{-mod-}7y \Leftrightarrow (\exists z)(0\text{-mod-}7z\ .\ xALMLT_{mod\text{-}7}y,y,z\ .\ yALMLT_{mod\text{-}7}z,x,y))$$

For any things, x and y, x is 2-mod-7 and y is 1-mod-7 if and only if there is at least one thing, z, such that

i) z is 0-mod-7,

ii) x is at least as much larger, mod-7, than y as y is, mod-7, than z, and

iii) y is at least as much larger, mod-7, than z as x is, mod-7, than y.

The predicate 1-mod-7 is basic to 1-mod-7 successions of strings from one string-pair to another string-pair, and x, y, and z correspond to the string-interval analogy 2:1::1:0 and 1:0::2:1.

(x)(3-mod-7x ⇔

$$(\exists y)(\exists z)(2\text{-mod-}7y\ .\ 1\text{-mod-}7z\ .\ x\mathrm{ALMLT}_{\text{mod-}7}y,y,z\ .\ y\mathrm{ALMLT}_{\text{mod-}7}z,x,y))$$

For any thing, x, x is 3-mod-7 if and only if there is at least one thing, y, and there is at least one thing, z, such that

i) y is 2-mod-7,

ii) z is 1-mod-7,

iii) x is at least as much larger, mod-7, than y as y is, mod-7, than z, and

iv) y is at least as much larger, mod-7, than z as x is, mod-7, than y.

The predicates 2-mod-7 and 3-mod-7 serve principally to specify string-interval sameness between different string-pairs, and x, y, and z correspond to the string-interval analogy 3:2::2:1 and 2:1::3:2. The predicate 'is at least as much larger, mod-7, than … as … is, mod-7, than' defines relationships among mod-7 string-numbers that are sufficient as a basis for the statistical account of the 35 Hurrian scores and the analysis of h.6 above. Nonetheless, it should be emphasized that the extension of such relationships consists of numbered strings rather than musical tones in any usual sense.

In the next segment of this study, i) a detailed scrutiny of U.7/80 shows how one can conclude that the numbered strings considered above correspond to tones in a non-degenerate well-formed (WF) cycle and ii) an examination of ways in which lyres and, especially harps, were depicted in Mesopotamia narrows the field of possible tunings for strings 1 to 7 to descending or ascending pitches in what might be termed, respectively, 'general diatonic' or 'general *pélog*,' albeit with the precise fundamental-frequency ratio of the octave regarded as open to relatively wide interpretation. As well, the next segment iii) shows how relationships of sameness, adjacency and analogy defined above can be regarded as counterparts to the Gestalt Grouping Principles of, respectively, Similarity, Proximity, and Common Fate, resulting in perceptual counterparts to the aspects of compositional design outlined in the present article.

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